Study on the Influence of Component Uncertainty on Reliability Estimation of Multi-State Systems with Continuous States

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Abstract. Existing reliability evaluation methods rely on the availability of accurate component states data. They will become ineffective when the states themselves are uncertain or unknown, which usually happens during the early stages of the development of new systems. In such cases it is important to understand how uncertainties will affect the system reliability measures. Another drawback of current methods studying reliability of Multi-State System (MSS) is that they only considered the systems whose components have several discrete states. For those whose components have continuous states, these methods are not effective either. This paper considered the continuous distribution of components states during the approximation of Multi-State System (MSS) reliability and proposed a method to assess the reliability of this kind of system using Monte-Carlo simulation. This method will also be useful when we have no enough data to know the exact discrete states and related probability, and can only estimate components states distribution types and related parameters. Two examples were employed to illustrate the method. Comparison of the two examples shows that component state uncertainty has significant influence on the assessment of system reliability. Our effort will make the reliability approximation more realistic compared with existing methods.

Key Words: uncertainty analysis, Multi-State System, system reliability, sensitivity, uncertainty, Monte Carlo simulation

Introduction

Recently the question of reliability has become a matter of great interest due to the increased competition, complex product design and development, the use of increasingly sophisticated manufacturing processes, particularly in the area of defence and space technology, and increasing focus on customer satisfaction. Although a great deal of progress has been made, many shortcomings still exist in this field. For instance, existing classical reliability prediction methods may not be effective enough to provide the required accuracy particularly during the product development process (Wong, 1990). The greatest uncertainty associated with reliability predictions is the variability or non-deterministic nature of the distribution parameters. Sometimes the information available on the failure characteristics of a product may not be well-defined and known. This is particularly true during the early stages of the development of new products. Unfortunately, this problem has not been addressed completely despite tremendous amount of efforts by the researchers over a long period of time (Bhamare, Yadav, and Rathore, 2007). People usually use point estimates and plug a number as the value of the unknown parameter into the model to compute the reliability. This has not been a very effective way to deal with uncertainty because we have no idea how the parametric uncertainties propagate into final measures such as reliability, performance and deformability (Yin et al., 2001). Therefore, analysis and quantification of uncertainty around the parameters play an important role in making reliability assessment realistic.

Besides the problem with parametric uncertainty during reliability assessment, traditional reliability assessment has been based on binary-state systems. System reliability in a binary-state context is the probability that the system properly provides the service for which it was intended, under the condition that the system and its components can be either fully working or failed (completely nonworking) (Ramirez-Marquez and Coit, 2005b). Numerous approaches and methodologies have been proposed to solve this difficult problem.

However, researchers have indicated that in some cases, binary state theory fails to characterize the actual system reliability behaviour, which is multi-state (Ramirez-Marquez and Coit, 2005a). For systems such as water distribution, telecommunications, oil and gas supply, and power generation & transmission, some components of the system may be operating in a degraded state causing the system to provide service at less than full capacity. However, the system may still be able to provide an acceptable level of service, or perhaps, a partial level of service. Such systems are called Multi-State Systems (MSS). For MSS, reliability can be defined as the probability that the system capacity can meet a required demand when the system components and demand follow a multi-state behaviour (Ramirez-Marquez and Coit, 2005b). Many MSS reliability models have been proposed to describe such systems focusing on modelling and analysis of reliability.

Existing MSS models mainly considered two general types of multi-state behavior (Ramirez-Marquez and Levitin, 2008). The first one involves a system where components are binary but different kinds of components have different nominal performance levels leading the system to work at different demand levels. These systems are usually known as MSS with binary-capacitated components (MSBC). The second version considers multi-state components so that system performance is dictated by the states of the components. In short, existing MSS models assume that components have only several (two or more) discrete states and each state has a particular probability to happen.

However, there are also some other systems with more complicated behavior. States of these systems and their components may be continuous instead of discrete. That is, system and component states can be any value within an interval. For these systems, current MSS model will not be effective to estimate the reliability.

In some conditions, we may know the several states and their probability a component or system may take. In these cases, existing methods (Marquez & Coit, 2004; Lin, 2001; Yeh, 2004;) can be used to effectively estimate system reliability. Under some other conditions, however, we may not know what states and the corresponding probability each component may take. We may just know some parameters (e.g., mean, maximum or minimum) and the probability distribution type of each component state. In these cases, existing methods (Marquez & Coit, 2004; Lin, 2001; Yeh, 2004;) will not be effective to estimate the system reliability either.

This paper has two main targets, one is to propose a method to approximate reliability of MSS with components that have continuous capacity and the other is to illustrate the effect of component uncertainty may have on the system reliability.

Assumptions:

- Component states are statistically independent.
- Demand loads during associated time intervals are known and fixed.

Literature Review

Previous studies on system reliability are mainly on binary-state systems. Ever since researchers realized the existence of multi-state systems, reliability analysis of multi-state systems has received considerable attention. Theoretical and applied studies have been devoted to the areas of multi-state system reliability, such as simulation, approximation methodologies, and optimization.

Levitin, et al. (1998) generalized the redundancy optimization problem to multi-state systems and proposed a procedure which determines the minimal-cost series-parallel system structure subject to a multi-state availability constraint. They also developed a fast procedure, based on universal generating function, to evaluate the multi-state system availability. Ramirez-Marquez and Levitin (2008) proposed an effective approach for the estimation of reliability confidence bounds based on component reliability and uncertainty data for multi-state systems with binary-capacitated components using the universal generating function technique. The universal generating function has proven to be a valuable and efficient tool for relatively complex systems.

Lin (2001) extended stochastic-flow network model of binary-state network to multi-state network to compute the exact multi-state system reliability. The algorithm of Lin (2001) introduced the method used to compute minimal path sets of binary-state systems to multi-state systems. However, they limited the states of system components to be weakly homogeneous. That is, components can have a different number of states, yet for any two components h and k with bh =lh, bk=lk, and lh >lk, the first lk component states must be equal. Thus, the methodology may not be suitable for systems whose components are heterogeneous.

Yeh (2004) proposed a minimal cut sets approach to evaluate the reliability in terms of MCs in a stochastic-flow network. The approach is an extension of the best of known algorithms for solving the d-MC (a special MC but formatted in a system-state vector, where d is the lower bound points of the system capacity level) problem from the stochastic-flow network without unreliable nodes to that with unreliable nodes by introducing some simple concepts. Besides the limitation that components are weakly homogeneous, this model also limited that the capacity of component must be an integer-valued random variables.

Ramirez-Marquez and Coit (2005a) described a Monte-Carlo (MC) simulation methodology for estimating the reliability of a multi-state network. Within their model, components of the network follow a degradation pattern that reduces the ability of the system to provide some required service, which makes the network and its components all have multiple states. They proposed an information sharing approach, that is, a selected number of MMCV called offspring cuts inherit information from a select number of MMCV called parent cuts. This approach significantly reduced the number of vector enumerations needed to obtain all MMCV and computation load.

As we reviewed above, existing models on MSS assume that components have only several discrete states and the probability of each state to happen is also known. These models will not be useful for systems whose states are continuous instead of discrete. They will not be useful either when we do not know the states and their corresponding probability a component may have. This paper will propose a method using Monte-Carlo simulation to approximate MSS reliability considering continuous component states and will also illustrate the influence that uncertainty of component states may have on the estimation of system reliability.

Reliability Estimation

Background. Reliability assessment is to assess the probability that a system will perform a required function without failure under stated conditions for a stated period of time. In a reliability prediction analysis, the components of a product or system are studied in an effort to predict the rate at which the product or system will fail.

As discussed previously, most of the studies on reliability assessment are based on the assumption that the system is a binary-state system; some other studies on multi-state systems are based on the assumption that the system components have several discrete states. For systems where binary-state analysis is insufficient, incorrect reliability assessment can lead to faulty decision-making regarding system performance. Unnecessary expenditures, incorrect maintenance scheduling, and reduction of safety standards can potentially be related to unsatisfactory reliability assessments (Ramirez-Marquez & Coit, 2005b). If the models on multi-state systems that have discrete component states are used to assess the reliability of systems that have continuous components, similar problems may happen.

MC simulation. For MSS, whenever no economic or time constraints exist, an effective approach to obtain the true value of their reliability would be to test an infinite number of MSS in real-life situations until failure occurs. Unfortunately, MSS and component testing is limited to tight economic budgets and schedules. Thus, it is often unrealistic and infeasible to perform extensive system testing both at the component and system level. A more efficient approach to estimate MSS reliability is Monte-Carlo (MC) simulation (Ramirez-Marquez and Levitin, 2008).

Lawless (1982) outlined the theory of MC simulation application. The MC method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. It is particularly useful for approximating stochastic relationships when no known exact expression is available (Wasserman, 2003). MC methods have proven efficient in statistical analysis of uncertainty in reliability engineering-related problems. Often it is found useful for complex reliability model which is difficult to handle by any of the available analytic methods (Bhamare, Yadav and Rathore, 2007). The MC has the capability of handling practically every possible case regardless of its complexity with a relatively simple mathematical formulation

(Papadrakakis and Lagaros, 2002). The proposed method will use Monte-Carlo (MC) simulation to generate state vectors at each simulation run based on components state probability distribution.

Simulation Procedure. In the proposed methodology, at each run of the simulation, a number for each component will be generated to depict its capacity based on the state probability distribution using Monte-Carlo. All the capacity number will compose a state vector of the system for the particular run.

Once this state vector has been generated, the maximum capacity can be computed using the minimum-cut maximum flow theorem. The maximum capacity will be the state of the system at the particular run.

So, for each simulation run, a state vector of the system components and the capacity of the system can be generated. After all the simulation runs, we can get the capacity probability distribution of the system. Based on the capacity distribution, we can get the probability that the capacity of the system is no less than the demand d, which will be the system reliability.

Then we can analyze the influence that uncertainty of component parameters have on the system state. Do sensitivity analysis to see how much the changes of the component state distribution parameters will affect the state distribution of the whole system reliability.

Illustrative Case Study

In this section, two examples are employed to illustrate the proposed method. We will compare the results of the two examples to see the effect parameter uncertainty will have on the system reliability approximation. Both of the two examples are about the same system depicted as a network in Fig. 1. This network is called the ARPA and is frequently used as an illustrative example in studies on binary state reliability (Jin & Coit, 2003). The ARPA network has loose connectivity and is presented here to illustrate the influence that uncertainty of component state distribution parameters may have on the state of the assessment of the reliability of the whole system.

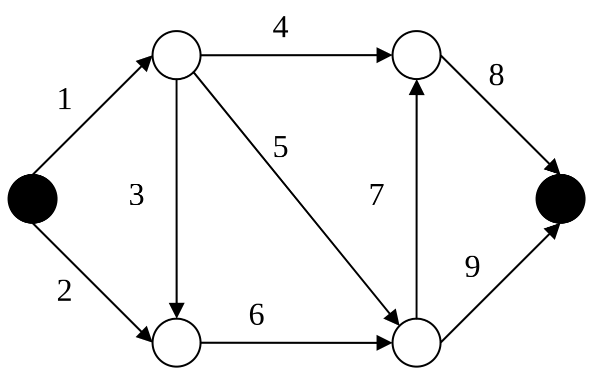

Figure 1 ARPA network

Each component of the network in both examples has the same mean, maximum and minimum capacity. The difference is that they have different probability distribution. Capacity of components in the first example has lognormal distribution and that of the second example has Normal distributions, as shown in table 1 and table 2. To keep the capacity of each component within the interval of its minimal and maximal capacity, we did a little modification to the distribution during the simulation. If the state is larger than maximum capacity, let it equal to maximum capacity. If the state is less than minimal capacity, let it equal to minimal capacity. It is desired to obtain the reliability of system demand d=8.8 under each condition. That equals to the probability that the system demand is greater than or equal to d=8.8 according to the definition of MSS reliability definition (Ramirez-Marquez and Coit, 2005b).

$$R = P_r\{d \geq 8.8\}$$

We used *Crystal Ball* together with *MS Excel* to do the simulation for 2000 trials separately. From figure 2 and figure 4, it can be seen that the system reliability is easy to obtain. It can be found from figure 2 that if the components states follow lognormal distribution, the reliability of the network will be 97.83%. From figure 4, we found that if the components states follow normal distribution, the reliability of the network will be 96.96%.

We can find the influence of different types distribution may have on the system reliability based on the comparison of the two examples. From table 1 and table 2, we know that the two examples have the same parameter values. If we use existing point estimation methods, we will get the same results about the reliability. However, the reliability of the two examples is not the same when we consider probability distribution using the methods this paper proposed. Although from figure 3 and figure 5, we can see that the difference between the mean (9.88 and 9.90 separately) and the maximum (11.74 and11.89 separately) are not so significant, the difference between system reliability (97.83% and 96.96% separately) is significant according to figure 2 and figure 4.

Table 1: Lognormal capacity distribution

Arc	Maximal Capacity	Probability distribution	Parameters		
			Location	Mean	Stand Deviation
1	10	Lognormal	0	8	0.8
2	6	Lognormal	0	5	0.5
3	5	Lognormal	0	4	0.4
4	3	Lognormal	0	2	0.2
5	4	Lognormal	0	3	0.3
6	7	Lognormal	0	5	0.5
7	6	Lognormal	0	5	0.5
8	9	Lognormal	0	7	0.7
9	5	Lognormal	0	4	0.4

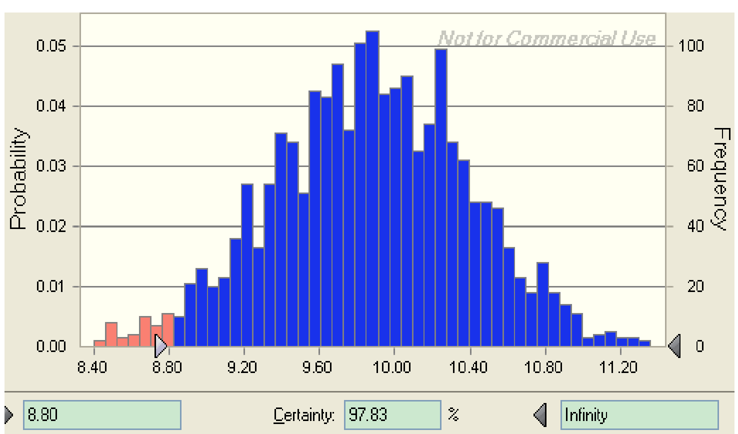

Figure 2

2,000 Trials Statistics View

Statistic	Forecast values
Trials	2,000
Mean	9.88
Median	9.88
Mode	---
Standard Deviation	0.53
Variance	0.28
Skewness	0.0323
Kurtosis	3.10
Coeff. of Variability	0.0534
Minimum	8.31
Maximum	11.74
Mean Std. Error	0.01

Figure 3

Table 2: Normal capacity distribution

Arc	Maximal Capacity	Probability distribution	Parameters		
			Location	Mean	Stand Deviation
1	10	Normal	0	8	0.8
2	6	Normal	0	5	0.5
3	5	Normal	0	4	0.4
4	3	Normal	0	2	0.2
5	4	Normal	0	3	0.3
6	7	Normal	0	5	0.5
7	6	Normal	0	5	0.5
8	9	Normal	0	7	0.7
9	5	Normal	0	4	0.4

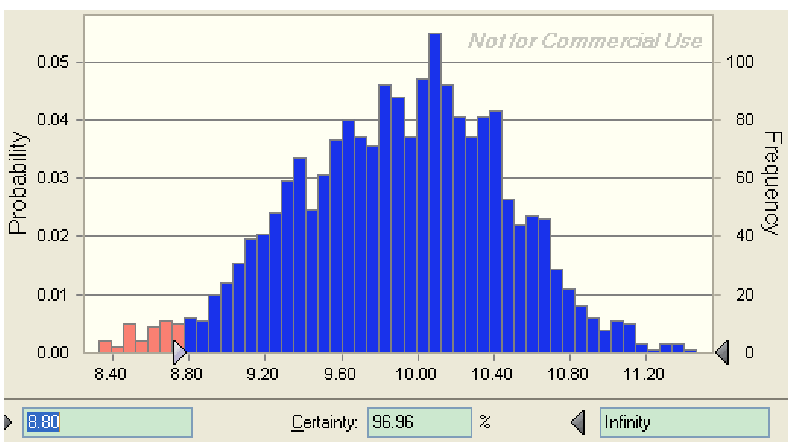

Figure 4

2,000 Trials Statistics View

Statistic	Forecast values
Trials	2,000
Mean	9.90
Median	9.93
Mode	---
Standard Deviation	0.56
Variance	0.31
Skewness	-0.2407
Kurtosis	3.14
Coeff. of Variability	0.0567
Minimum	7.58
Maximum	11.89
Mean Std. Error	0.01

Figure 5

Conclusions and Limitations

This paper considered the continuous distribution of component capacity during the approximation of Multi-State System (MSS) reliability. We also proposed a method to assess the reliability of this kind of system. This method will be useful to assess the reliability of systems that have components with continuous capacity. It will also be useful when we do not have enough data to know the multiple discrete states and related probability of a system, and can only approximate its distribution and related parameters. Comparison of the two examples with same parameter and different distribution shows that component state uncertainty will have significant effect on the system reliability.

Besides the assessment of system reliability, we may also concern about how to improve it. In order to find the effective way to improve system reliability, it is important to know the importance of each component. A lot of studies have been done on the assessment of Component Importance (CI). However, these studies only considered concrete component states. It is also needed to assess CI considering continuous component states.

Reference

Bhamare, S. S., Yadav, O. P. and Rathore, A. 2007. Evolution of reliability engineering discipline over the last six decades- a comprehensive review. Int. J. Reliability and Safety. 1(4):

Dylis, D.D. and Priore, M.G. 2001. A comprehensive reliability assessment tool for electronic system. Proceedings of Annual Reliability and Maintainability Symposium.

Griffith, W. 1980. Multi-state reliability models. J. Appl. Prob. 17: 735–744.

Jensen, F. 1995. Reliability: next generation', Microelectronic Reliability. 35: 1363–1375.

Papalambros, P.Y. and Wilde, D.J. 1991. Principles of optimal design: modeling and computation. New York: Cambridge University Press.

Pecht, M. and Dasgupta, A. 1996. Physics-of-failure: an approach to reliable product development. IRW Final Report, IEEE.

Rao, S.S. 1996. Engineering Optimization: Theory and Practice. New Delhi: New Age International Publishers.

Snook, I., Marshall, J.M. and Newman, R.M. 2003. Physics of failure as an integrated part of design for reliability. proceedings of Annual Reliability and Maintainability Symposium. 46–54.

Natvig B, Somo S, Holen A, Hoasen G. 1986. Multi-state reliability theory—a case study. Adv Appl Probab. 18:921–32.

Jin T and Coit D. 2003. Network reliability estimates using linear and quadratic unreliability of minimal cuts. Reliab Eng Syst Saf. 82(1):41–8.

Kuo, S., Lu, S. and Yeh, F. 1999. Determining terminal pair reliability based on edge expansion diagrams using OBDD. IEEE Trans. Reliab. 48(3): 234–246.

Kettelle, J.D. 1962. Least-cost allocation of reliability investment. Operations Research. 10(2).

Levitin, G., Lisnianski, A., Ben-Haim, H., and Elmakis, D. 1998. Redundancy optimization for series-parallel multi-state systems. IEEE Trans. Reliab. 47(2): 165–172.

Levitin, G. and Lisnianski, A. 2003. Multi-State System Reliability. Series on Quality, Reliability and Engineering Statistics. 6: World Scientific Publishing,

Lin, Y. 2001. A simple algorithm for reliability evaluation of a stochastic-flow network with node failure. Comput. Oper. Res. 28: 1277–1285.

Lin, Y. 2002. Using minimal cuts to evaluate the system reliability of a stochastic- flow network with failures at nodes and arcs. Reliab. Eng. Syst. Saf. 75: 41–46.

Levitin, G. 2004. A universal generating function approach for the analysis of multi-state systems with dependent elements. Reliab. Eng. Syst. Saf. 84: 285.

Ramirez-Marquez, J. E. and Coit, D. 2003. Alternative approach for analyzing multi-state network reliability. Industrial Engineering Research Conf. (IERC), Portland.

Ramirez-Marquez, J. E., Coit, D., and Tortorella, M., 2003. Multi-state two-terminal reliability—a generalized cut set approach. IEEE Trans. Reliab.

Ramirez-Marquez, J. E. and Coit D. W. 2005a. A monte-carlo simulation approach for approximating multi-state two-terminal reliability.

Ramirez-Marquez, J. E. and Coit, D. W. 2005b. Composite Importance Measures for Multi-State Systems with Multi-State Components. IEEE TRANSACTIONS ON RELIABILITY. 54(3)

Ramirez-Marquez, J. E. and Coit, D. W. 2007. Multi-state component criticality analysis for reliability improvement in multi-state systems. Reliability Engineering and System Safety. 92: 1608–1619.

Ramirez-Marquez, J. E. and Levitin, G. 2008. Algorithm for estimating reliability confidence bounds of multi-state systems. Reliability Engineering and System Safety. 93: 1231–1243

Vasseur, D. and M. Llory. 1999. International survey on PSA figures of merit. *Reliab. Eng. Syst. Saf.* 66: 261–274,

Wong K.L. 1990. What is wrong with the existing reliability prediction methods? Quality and Reliability Engineering International. 6: 251–257

Wasserman, G.S. 2003. Reliability Verification, Testing, and Analysis in Engineering Design. New York: Marcel Dekker Inc.

Yeh, W. 2004. A simple MC-based algorithm for evaluating reliability of a stochastic-flow network with unreliable nodes. Reliab. Eng. Syst. Saf. 83(1): 47.

Yin, L., Smith, M.A.J. and Trivedi, K.S. 2001. Uncertainty analysis in reliability modeling. Proceedings Annual Reliability and Maintainability Symposium.

Zio, E. and Podofillini, L. 2003. Monte-Carlo simulation analysis of the effects on different system performance levels on the importance on multi-state components. Reliab. Eng. Syst. Saf. 82: 63–73.