134 October 14, 1985

C. Wayne Proctor Attendance: Proctor, Ingram, Lelek, Vobach, West.

Wayne Proctor presents some results in Class W continua. A characterization of absolutely C^*-smooth continua due to Grispolakis, Tymchatyn and Nadler leads to the following definition. A continuum M is continuously ray extendable provided, for each continuous mapping $f: X \to M$ of a continuum X onto M and for each ray L such that $L \cup M$ is a compactification of L with remainder M, there exists a ray (closed half-line) R, with $R \cup X$ a compactification of R and X its remainder, such that f can be extended continuously to a mapping of $R \cup X$ onto $L \cup M$.

Theorem 1. Each continuously ray extendable continuum is in Class W.

Theorem 2. Each chainable continuum is continuously ray extendable.

It is unknown whether any of these two theorems can be reversed. Both cannot be reversed, since there are Class W continua which are not chainable. Also, the following question is suggested by Theorem 2:

✓ Problem (W.T. Ingram). Is each continuum of span zero continuously ray extendable?

135 November 4, 1985

Attendance: Ingram, Lelek, Slye, Gehrke, Mouiken, West.

B. Mouiken presents some results of L. Rubin on strongly infinite-dimensional spaces.

Theorem 1. Every strongly infinite-dimensional compact metric space contains a G_δ-subspace which is totally disconnected and hereditarily strongly infinite-dimensional.

[Hereditary strong infinite-dimensionality means that subsets are either empty, or 0-dimensional, or strongly infinite-dimensional.] This theorem completes previous results of Rubin's and other authors, where different types of hereditarily strongly infinite-dimensional spaces were constructed (cf. Proc. Amer. Math. Soc., vol. 79, 1980, pp. 153-154).

(cf. p. 10)

Theorem 2. If K is any strongly infinite-dimensional compact metric space, then K contains a totally disconnected subspace homeomorphic to a hereditarily strongly infinite-dimensional closed subset of the space constructed in Theorem 1 for the Hilbert cube.

(cf. p. 4)

The concept of essential families plays an important role in these theorems, not only as a device to define strong infinite-dimensionality but also as a method of construction. The asymmetry in Theorem 1 gives rise to the following:

Problem (A. Lelek). Can the assumption in Thm. 1 that the space is compact be replaced by the assumption that it is an absolute G_δ-space?

Vrunda Prabhu

(136) November 18, 1985

Attendance: Ingram, Lelek, Mouken, Prabhu, West.

B. Mouken continues her presentation of Rubin's results.

Example. There exist a weakly infinite-dimensional compact metric space $\mathbb{P}$ which contains a dense G_δ-subspace $\mathbb{Y}$ such that $\mathbb{Y}$ is totally disconnected, hereditarily strongly infinite-dimensional, and $\mathbb{P} \setminus \mathbb{Y}$ is not countable dimensional.

√

The space $\mathbb{P}$ is a continuous image of a closed subset ~~of~~ an (arbitrary) hereditarily strongly infinite-dimensional compact metric space. The existence of such a space $\mathbb{P}$ completes some previous results of R. Pol (cf. Proc. Amer. Math. Soc., vol. 82, 1981, pp. 634-636). The total disconnectedness of $\mathbb{Y}$ is used to show that $\mathbb{P}$ is weakly infinite-dimensional.

√

Eugene M. Isrll
Michael F. Granado

(137) April 18, 1986

Attendance: Krasinkiewicz, Cook, Lelek, Insall, Granado, West, Ingram.

J. Krasinkiewicz presents essential mappings into products of manifolds. A mapping $f: X \to I^n$ is called essential provided the restriction

$$f|f^{-1}(\mathrm{bd}\, I^n): f^{-1}(\mathrm{bd}\, I^n) \to \mathrm{bd}\, I^n \approx S^{n-1}$$

is not extendable over X. All spaces are assumed to be metrizable.

Let M be a compact connected manifold of dimension ≥ 1. If $X \xrightarrow{f} M$, then a mapping $X \xrightarrow{g} M$ is said to be an <u>admissible deformation</u> of f provided there exists a homotopy

$$H : (X, f^{-1}(\partial M)) \times I \to (M, \partial M)$$

such that $H_0 = f$ and $H_1 = g$. If, in addition, $H(x,t) = f(x)$ for $x \in f^{-1}(\partial M)$, then g is called a <u>∂-deformation</u> of f.

Let $f = \{f_j\} : X \to \prod_{j \in J} M_j$ be a mapping of X into a product of manifolds. Another mapping $g = \{g_j\}$ is an admissible deformation of f provided g_j is an admissible deformation of f_j for each $j \in J$. Similar definition is used for ∂-deformations.

A mapping $f : X \to \prod_{j \in J} M_j$ is <u>essential</u> provided every admissible deformation of f is onto. This definition agrees with the definition of essential mappings $f : X \to I^n$, and, in the case of one manifold, it was used by J. Grispolakis and E.D. Tymchatyn. A mapping $f = \{f_j\} : X \to I^n$ is essential in this sense if and only if the family of pairs

$$(f_1^{-1}(0), f_1^{-1}(1)),\ (f_2^{-1}(0), f_2^{-1}(1)),\ \dots,\ (f_n^{-1}(0), f_n^{-1}(1))$$

is essential in the sense that each collection of n cuttings of X between sets in those pairs has a point in common. The space X is <u>strongly infinite-dimensional</u> (SID) iff there exists an infinite countable essential family $\{(A_j, B_j)\}_{j \in J}$ in X. It is known that X is SID iff there exists an essential mapping $X \to I^\infty$.

$\mathring{M} = M \setminus \partial M$

<u>Theorem 1.</u> Let $f: X \to M_J = \prod_{j \in J} M_j$ be a mapping and let A be a closed subset of X. Then, for any choice of $y_j \in \mathring{M}_j$, $j \in J$, there exists a neighborhood V_j of y_j in M_j and a ∂-deformation g of f such that $g(A) \cap \prod_{j \in J} V_j = \emptyset$.

<u>Theorem 2.</u> Let $f: X \to M_J$ and let $N_j \subset M_j$ be a submanifold with $\dim N_j = \dim M_j$ for $j \in J$. Then f essential implies that the restriction of f to

$$f^{-1}\left(\prod_{j \in J} N_j\right) \longrightarrow \prod_{j \in J} N_j$$

is essential.

(ca. 1938)
(cf. also Hopf, 1930)

<u>Problem 1</u> (K. Borsuk). Let $f: X \to M$ be an essential mapping and let N be a submanifold of M. Is the mapping $f^{-1}(N) \to N$ determined by f essential?

<u>Theorem 3.</u> If the restriction of $f: X \to M_J$ (is not essential) to a subset Y of X), then the restriction of f to some neighborhood of Y in X is not essential.

<u>Theorem 4.</u> Let $f = \{f_{J_k}\}: X \to \prod_{k \in K} M_{J_k}$ and let $(A_k)_{k \in K}$ be a covering of X such that no restriction $f_{J_k} | A_k : A_k \to M_{J_k}$ is essential. The f is not essential. [This is proved using Thms. 3&1

<u>Theorem 5</u>. Every homeomorphism $X \twoheadrightarrow M_J$ is an essential mapping.

Brouwer's fixed point theorem can be restated as that the identity mapping $I^n \xrightarrow{id} I^n$ is essential. Thus Thm. 5 is a generalization of this.

A mapping $f: X \to I$ of a compact space X is essential iff there exists a continuum in X meeting both $f^{-1}(0)$ and $f^{-1}(1)$.

<u>Theorem 6</u>. If $f_j: X_j \to I$ are onto mappings of continua X_j onto the unit interval I $(j \in J)$, then the product mapping $\prod_{j \in J} f_j : \prod_{j \in J} X_j \to I^J$ is essential.

The answer to Problem 2 is <u>no</u>: there exists a connected space (Robert's example based on Erdös' example in dimension theory) X such that $\dim X = \dim X \times X = 1$. (Krasinkiewicz, 6/19/86)

<u>Problem 2</u>. Is Thm. 6 true if continua X_j are replaced by connected spaces?

<u>Problem 3</u>. Let $f: X \to M_J$ be an essential mapping and let $h: M_J \twoheadrightarrow N_K$ be a homeomorphism between two products of manifolds. Must hf be essential? The answer is "yes" in the cases:

(a) M_J is a product of closed manifolds,
(b) M_J is a manifold (i.e., J is finite),
(c) h is a product of homeomorphisms,
(d) M_J and N_K are products of cells.

[Universal mappings, in the sense of W. Holsztyński, are used to prove case (d).]

<u>Theorem 7</u>. A mapping f into ~~the~~ a product of cells

is essential iff f is universal.

Corollary. Let $f, g : X \to \prod_{j \in J} Q_j$, where Q_j is a cell ($j \in J$). Then f restricted to the set $\{x \in X : f(x) \neq g(x)\}$ is not essential.

Let $f : X \to M_J$ be a mapping onto a product of manifolds. A set $Y \subset X$ is said to be a membrane of f provided $f|Y$ is essential. A set $S \subset X$ is said to be a separator of f provided $X \setminus S$ is not a membrane. A subset A of X is a membrane (separator) of f iff A meets each closed separator (resp., membrane) of f. Every separator of a mapping contains a closed separator.

(138) May 2, 1986

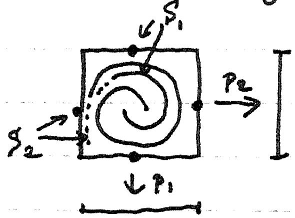
Attendance: Krasinkiewicz, Cook, Lelek, West, Granado, Ingram.

J. Krasinkiewicz continues his presentation of essential mappings.

A membrane is an irreducible membrane provided no proper closed subset of it is a membrane. Each irreducible membrane is connected. If X is compact and f is essential, then there exists an irreducible membrane of f. A subset $T \subset X$ is called a near-separator provided each neighborhood of T in X is a separator.

<u>Theorem 8.</u> Let $f=(f_{J_k}): X \to \prod_{k\in K} M_{J_k}$ be a mapping into the product of products of manifolds. Let S_k be a <u>closed</u> near-separator (separator) of f_{J_k} for each $k \in K$. Then $\bigcap_{k\in K} S_k$ is a near-separator (resp., separator) of f.

<u>Example.</u> The assumption of closedness is needed, as shown in these two spirals with two limits (each) added:



$S_1 \cap S_2 = \emptyset$

$(p_1, p_2) = id_{I^2}$

S_1, S_2 are near-separators

[Duality Thm.]

<u>Theorem 9.</u> Let $(f_J, f_K): X \to M_J \times M_K$. Then every near-separator S of f_K is a membrane of f_J. Furthermore, every neighborhood of S in X contains a closed membrane of f_J.

<u>Theorem 10.</u> If $X \to M_J$ is essential, then $\dim X \geq \dim M_J$.

<u>Corollary.</u> Let $(f_J, f_K): X \to M_J \times M_K$ be essential. If a set $Y \subset X$ meets every closed membrane of f_K, then Y is a membrane of f_J. Furthermore, every neighborhood of Y in X contains a closed membrane of f_J.

From this it follows that separating sets of R^n have essential mappings into S^{n-1} (Borsuk's Thm.).

(139) May 16, 1986

Attendance: Krasinkiewicz, Ingram, Lelek, West, Henderson.

J. Krasinkiewicz continues his presentation.

Problem 1. Let $A \subset R^n$ and $\dim A \leq n-2$. Is it true that every two points in $R^n \setminus A$ can be joined by a 1-dimensional continuum contained in $R^n \setminus A$?

✓ Problem 2. Let $A \subset R^n$. Does there exist a set $B \subset R^n$ such that $A \subset B$, $\dim B = \dim A$ and $\dim (R^n \setminus B) = n - \dim B - 1$?

An affirmative solution of Problem 2 would imply an affirmative solution of Problem 1.

We say that a manifold M $\subset X$ is k-flat in X provided there exists an embedding $h: M \times I^k \to X$ such that $M = h(M \times \{t\})$ where $t \in \mathring{I}^k$.

Theorem 11. Let $A \subset X$, $\dim A \leq k$, M $(k+1)$-flat manifold in X, and $B \subset M \setminus A$ a compact subset. Then, for every $\epsilon > 0$, there exists a continuum $Y \subset X \setminus A$ and a mapping $f: Y \to M$ such that:

[Thm. 11 generalizes Mazurkiewicz's thm. about $(n-2)$-dim. subsets of R^n.]

(i) Y is an irreducible membrane of f,

(ii) $f^{-1}(B) = B$ and $f(x) = x$ for $x \in B$,

(iii) $\rho(x, f(x)) < \epsilon$ for $x \in$ ~~[illegible]~~ Y.

Define collections $O(n)$ of spaces inductively as follows:

$O(-1) = \{\emptyset\}$,

$O(n)$ consists of spaces X such that, for every pair of disjoint closed sets A, B in X and every 0-dimensional set Z in X, there exists a cutting L of X between A and B such that $L \cap Z = \emptyset$ and $L \in O(n-1)$. $[\underbrace{X \in O(0)} \equiv \underbrace{\mathrm{Ind}\, X = 0}]$

The answer to Problem 3 is no: $R^2 \notin O(2)$. Also, the answer to the second part of Problem 4 is no. It is still unknown if all regular curves belong to $O(1)$.

(Krasinkiewicz & Lelek, 5/29/86)

There exists a regular curve $\notin O(1)$.

(Lelek & Tymchatyn, 4/10/89)

Problem 3. Does R^n belong to $O(n)$?

Problem 4. What curves belong to $O(1)$? In particular, do rational continua belong to $O(1)$?

Theorem 12. Let X be a compactum, let $f: X \to M_J$ be essential, and let $J = J_0 \cup J_1 \cup \ldots$ with $J_n \neq \emptyset$ and $J_m \cap J_n = \emptyset$ for $m \neq n$. Then there exists a continuum $Y \subset X$ such that:

(i) Y is a membrane of f_{J_0},

(ii) any positive-dimensional subset of Y is a membrane for one of the mappings f_{J_n}.

(cf. p. 2)

[This theorem produces examples of hereditarily strongly infinite-dimensional objects.]

The proof of this theorem becomes somewhat easier if, in condition (ii), one only considers non-degenerate subcontinua of Y instead of all positive-dimensional subsets. Pairs (A_n, B_n) of disjoint closed subsets of X are taken such that each non-degenerate subcontinuum of X meets both A_n and B_n for some $n = 1, 2, \ldots$

Corollary. There exists a continuum Y such that every positive-dimensional subset of Y admits an essential mapping onto every countable product of manifolds.

To show this, Theorem 12 is used together with a result of Cheeger and Kister (1970) that there are only $\aleph_0$ topological types of manifolds.

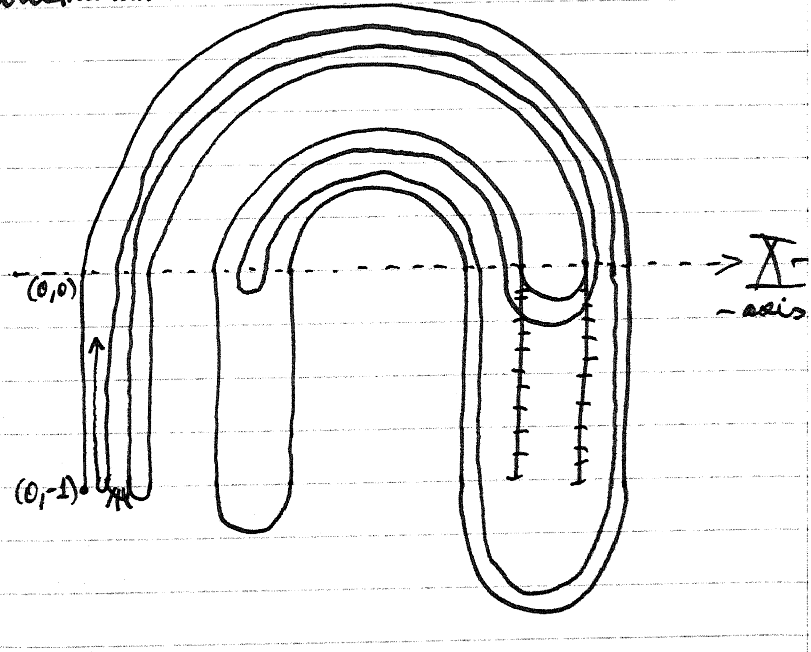
(140) July 18, 1986

Attendance: Davis, Ingram, Cook, Lelek, West.

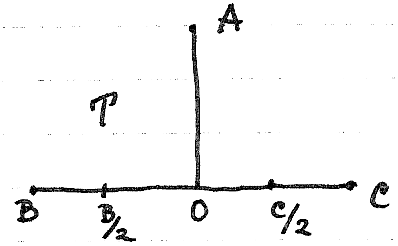
James F. Davis

J. Davis presents his results (joint with W. T. Ingram) about an atriodic tree-like continuum with positive span which admits a monotone mapping to a chainable continuum.

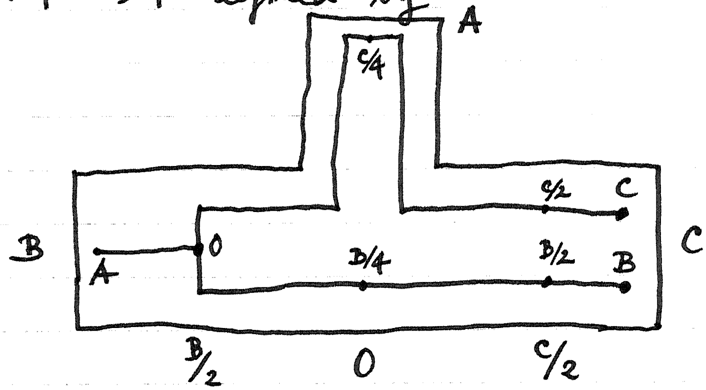
(Solves in the negative problems # 92 & 105 of UHMPB.)



Let T be a simple triod, and

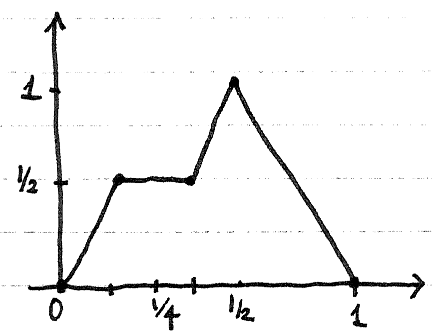


$f: T \to T$ defined by



(f)

together with $g: I \to I$ defined as

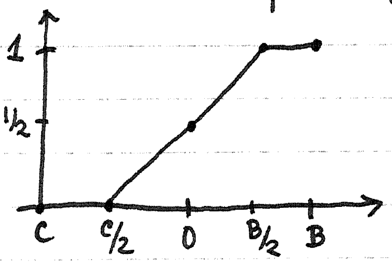


(g)

giving $X = \varprojlim\{X_n, f_n\}$, $Y = \varprojlim\{Y_n, g_n\}$, where $X_n = T$, $f_n = f$, $Y_n = I = [0,1]$, $g_n = g$. Then Y is the Brouwer-Janiszewski-Knaster continuum. There is a mapping $\mu: X \twoheadrightarrow Y$ defined by

$$\mu(x_1, x_2, \dots) = (h(x_1), h(x_2), \dots),$$

where $h: T \to I$ is defined by $h[0, +] = \frac{1}{2}$, and



(h)

[The diagram

$$\begin{tikzcd} T \arrow[d, "h"'] & T \arrow[l, "f"'] \arrow[d, "h"] \\ I & I \arrow[l, "g"] \end{tikzcd}$$

commutes.]

<u>Theorem 1.</u> The mapping μ is monotone, and the only non-degenerate point-inverse under μ is the arc

$$\mu^{-1}(0, 0, \dots) = \{(t, t, \dots) : t \in [C/2, C]\}.$$

<u>Theorem 2.</u> X is atriodic; even more, every proper subcontinuum of X is an arc.

✓ <u>Theorem 3.</u> $\sigma(X) > 0$. [This is done essentially by the use of Ingram's technique.]

[W.T. Ingram has confirmed the existence of this homeomorphism; 11/10/86.]

At this moment there is a quasi-proof that this X is homeomorphic to the continuum pictured on page 11. There are two modifications of that continuum: one with turning points converging to a Cantor set, and another one with turning points converging to an interval. In X, the turning points converge to ~~the~~ the two-point set $\{(0, 0), (0, -1)\}$.

Jack McBryde
Philip M. Steel
Carl Nelson
Chu Che-Chen
M. Cook
Mike Sawyer

141

November 10, 1986

<u>Attendance</u>: Ingram, G. Johnson, Lelek, Insall, McBryde, Steel, Nelson, M. Cook, Chu Che-Chen, Sawyer.

<u>W.T. Ingram</u> presents his results (joint with J.F. Davis) on continua with positive span. A proof of Theorem 3 (page 13) is given. The following theorem is used:

<u>Theorem</u>. Let $M = \varprojlim \{ X_i, f_i^j \}$ be an inverse limit of continua. If there exists a number $\varepsilon_0 > 0$ such that $\sigma f_1^n \geq \varepsilon_0$ for each $n = 1, 2, \ldots$, then $\sigma M > 0$.

Actually, a stronger result is proved, namely, the <u>symmetric span</u> is positive for the continuum constructed by Davis & Ingram. The above theorem also holds for σ meaning the symmetric span. In the definition of symmetric span, due to H. Cook, the continuum $C \subset \underline{X} \times \underline{X}$ is required to have the property that $\Phi(C) = C$, where $\Phi : \underline{X} \times \underline{X} \to \underline{X} \times \underline{X}$ is the homeomorphism

$$\Phi(x_1, x_2) = (x_2, x_1),$$

and thus implying that $p_1(C) = p_2(C)$ for the projections p_1 and p_2 of the product $\underline{X} \times \underline{X}$. Some solenoids have symmetric span zero but span positive.

Problem (W. T. Ingram). Does there exist a simplicial continuous mapping f of a tree into itself such that $\sigma(f^2) > 0$ and

$$\lim_{n\to\infty} \sigma(f^n) = 0 \ ?$$

(142) May 4, 1987

Attendance: Ingram, H. Cook, Golubitsky, Lelek, McBryde, Nelson, M. Cook, Granado, Moebes, Sawyer, V. Williams.

W. T. Ingram presents his results on periodic points and mappings of continua. A continua theory generalization of Sharkovsky's theorem is obtained, related to the fixed point property.

Theorem 1. Suppose M is a continuum (or a top. space) and $f: M \to M$ is a mapping. Suppose K is a subcontinuum of M such that $f(K) \supset K$. If

(1) every subcontinuum of K has the fixed point property, and

(2) every subcontinuum of $f(K)$ is in Class W,

then there is a point $x \in K$ with $f(x) = x$.

[S. Nadler has an example of a mapping $f: I^2 \twoheadrightarrow D$, where D is a disk containing I^2, such that f does not have a fixed point.]

<u>Theorem 2.</u> If $f: M \to M$ is a mapping and there exist subcontinua H and K of M such that

(1) every subcontinuum of K has the fixed point property,

(2) every subcontinuum of $f(H)$ is in Class W,

(3) $f(K) \supset H$,

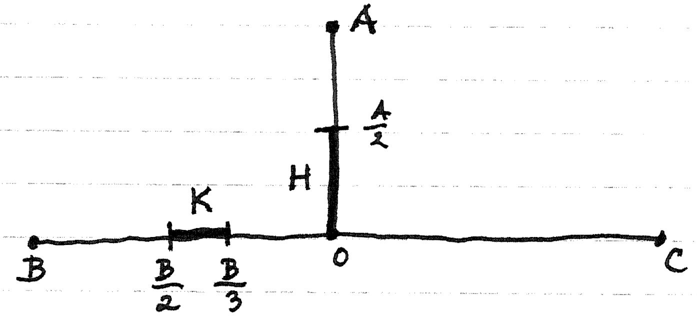
(4) $f(H) \supset H \cup K$,

(5) if $(f|H)^{-n}(K) \cap K \neq \phi$, then $n = 2$,

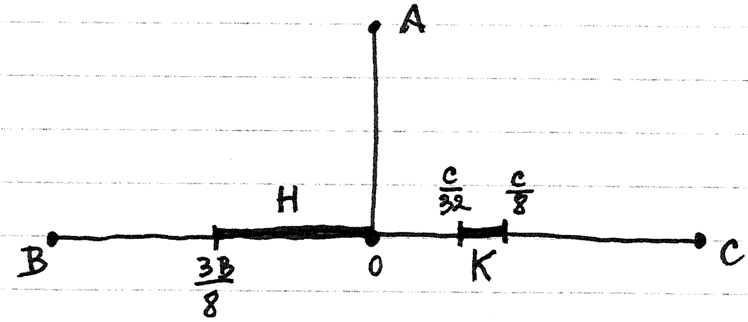
then K contains periodic points of f of every period.

[Conditions (1) and (2) are used to apply Theorem 1.]

Theorem 2 can be applied to Ingram's function on the simple triod:



and also to Ingram & Davis' function (see p. 12):



Theorem 3. Suppose $f: [0,1] \to [0,1]$ has a periodic point of prime period n and n is not a power of 2. Then $\varprojlim \{[0,1], f\}$ contains an indecomposable continuum. Moreover, for each i, there exists a mapping f_i with a periodic point of period 2^i such that $\varprojlim \{[0,1], f_i\}$ is hereditarily decomposable.

143 April 12, 1988

Attendance: M. Barge, H. Cook, Ingram, Lelek, M. Cook, ~~Simpson~~ Che-Chen, McBryde, Insall, Granado, Dounglove.

Marcy Barge presents prime ends and applications. Let $U \subset S^2$ be open, connected, and simply connected. If $F: S^2 \to S^2$ is a homeomorphism with $F(U) = U$ and the frontier of U has more than one point, then associated with U is the prime end compactification.

Example. There exists a homeomorphism of S^2 and an open, connected, simply connected region $U \subset S^2$ such that the rotation number on the circle of prime ends is irrational, and there does not exist a dense orbit.

Problem 1. If $F: S^2 \to S^2$ is a homeomorphism, $\Lambda \subset S^2$ is a continuum with $F(\Lambda) = \Lambda$, $S^2 \setminus \Lambda = U_1 \cup U_2$, U_1 and U_2 disjoint open, and

$\mathrm{Fr}\, U_1 = \Lambda = \mathrm{Fr}\, U_2$, and the prime end homeomorphism induced by F (from either side) is conjugate to an irrational rotation, then is it true that $F|\Lambda$ is <u>minimal</u>; that is, if $x \in \Lambda$, then $\{F^n(x) : n \geq 0\}$ is dense in Λ?

<u>Problem 2.</u> For what planar continua Λ is there a homeomorphism $F: \Lambda \to \Lambda$ such that F is minimal?

(144) November 7, 1988

<u>Attendance</u>: H. Kato, Ingram, Lelek, D. Brown, M. Cook, Chen Che-Chan, Darji.

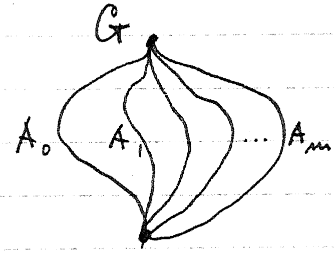
Hisao Kato
Udayan Darji

<u>H. Kato</u> presents results on geometric properties of Whitney continua. If $\omega: C(X) \to [0, \infty)$ is a Whitney map, the sets $\omega^{-1}(t)$, for $0 \leq t < \omega(X)$, are called <u>Whitney continua</u>. A topological property P is called a <u>Whitney property</u> if whenever X has P, so does every Whitney continuum $\omega^{-1}(t)$. ~~For example, are AR, LC^n, weakly chainable, Whitney properties?~~

<u>Theorem 1.</u> If G is a graph, ω is a Whitney map for G, then $\omega^{-1}(t)$ is a polyhedron for each t. [The proof of this result is the same as in R. Duda's papers in Fund. Math. #62, #63.]

Theorem 2. If G is a graph which contains at least one simple closed curve, ω is a Whitney map for G, $t_0 = \min\{\omega(S) : S$ simple closed curve in $G\}$, then $\omega^{-1}(t)$ is ~~homeomorphic to~~ of the same homotopy type as G for $0 \le t < t_0$. If G is a tree, then $\omega^{-1}(t)$ is of the same homotopy type as G for all t.

Theorem 3. Let G be the union of $(m+1)$ arcs with two common end-points:



Let ω be a Whitney map for G, and
$$t_0 = \max\{\omega(A_0 \cup \dots \cup A_{i-1} \cup A_{i+1} \cup \dots \cup A_m) : i = 0, \dots, m\}.$$
Then $\omega^{-1}(t)$ is of the homotopy type of the ~~space~~ S^m for $t_0 \le t < \omega(X)$.
[The proof of this result depends on a theorem of Lynch from PAMS 97.]

Theorem 4. Let $P = |K|$ be a connected polyhedron. There exists a Whitney map
$$\omega : C(|K^1|) \to [0, \infty)$$
such that $\omega^{-1}(t)$ has the homotopy type of P for some t (i.e., Whitney continua of graphs admit all homotopy types of compact connected ANR's).

K^1 = the 1-skeleton of K

Homotopy dimension and fundamental dimension, in the sense of shape theory, are also investigated for $\omega^{-1}(t)$.

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November 14, 1988

Attendance: Kato, Ingram, Lelek, Chu, M. Cook, Darji. ~~Kenton~~ Green

H. Kato continues his presentation of geometric properties of Whitney continua, for polyhedra P with $\dim P \geq 2$. The property of being locally

LC^0 connected is a Whitney property (S. Nadler). J. Krasinkiewicz & S. Nadler asked if the property of being LC^n or AR is a Whitney property. A. Petrus showed that there is a Whitney map ω for $C(D)$,

LC^2 D a 2-cell, such that $\omega^{-1}(t)$ is not LC^2 and not C^2 (2-connected).

LC^1 Theorem 1. Let X be a LC^1-continuum in a 2-manifold. Then $\omega^{-1}(t)$ is LC^1 for any Whitney map ω for $C(X)$. Moreover, if X is C^1, then $\omega^{-1}(t)$ is C^1. In particular, for any Whitney map ω for $C(D)$, D a 2-cell [so $C(D) \approx [0,1]^\infty$], $\omega^{-1}(t)$ is LC^1 and C^1.

It is known that if X is a 1-dimensional ANR, then $\omega^{-1}(t)$ are ANR's. [Lynch, BAPS 75, 1987.]

However, there exists a 2-dimensional AR Z (not contained in a 2-manifold) and a Whitney map ω for $C(Z)$ such that $\omega^{-1}(t)$ is not LC^1 and not C^1. This retract Z is the union of the dyadic solenoid and all the mapping cylinders of the bonding maps (an infinite telescope). The neighborhoods of points in the solenoid (in Z) are the unions of mapping cylinders for open sets, like Z itself.

Theorem 2. If X is a 1-dimensional continuum such that every proper subcontinuum of X is tree-like, then $Sh(\omega^{-1}(t)) = Sh\,X$.

Problem 1. Is "weakly chainable" a Whitney property?

Problem 2. If X is tree-like (or a dendroid), does $C(X)$ have the fixed point property?
[Case of "arc-like" was proved by J. Segal.]

(146) November 21, 1988

Attendance: Kato, H. Cook, Lelek, Ingram, Darji, M. Cook.

H. Kato presents his results on existence of expansive homeomorphisms. If X is a compact metric space, a homeomorphism $f: X \to X$ is called expansive provided there exists a number $c > 0$ such that, for any two points $x, y \in X$ $(x \neq y)$, there is an integer $n \in \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ with

$$d[f^n(x), f^n(y)] \geq C.$$

[The number C is called an expansive constant.]

It is known that there exist expansive homeomorphisms on:

$$\begin{cases} \text{the Cantor set}, \\ p\text{-adic solenoid } (p > 1), \\ n\text{-torus } T^n = \underbrace{S^1 \times \cdots \times S^1}_{n} \ (n \geq 2). \end{cases}$$

There do not exist expansive homeomorphisms on:

$$\begin{cases} \text{an arc}, \\ \text{the circle } S^1 \ (D^2 : 2\text{-cell}), \\ 2\text{-sphere } S^2 \ (D^3 : 3\text{-cell}). \end{cases}$$

Problem. Is it true that there do not exist expansive homeomorphisms on:

(1) any plane continuum,

(2) any tree-like continuum (in particular, any arc-like continuum),

(3) any Peano curve (i.e., a locally connected 1-dimensional continuum; in particular, Menger's universal curve)?

Theorem. There do not exist expansive homeomorphisms on:

(1) locally connected continua which possess a subset which is a dendrite with non-empty interior,

(2) uniformly arcwise connected continua,

(3) locally connected plane continua,

(4) dendroids.

147 February 20, 1989

Attendance: H. Cook, Ingram, Kato, Tymchatyn, Lelek, Slye, M. Cook, Finch.

M. Cook presents his result on periodic points for maps of the annulus. Let f be a mapping of a space into itself. Define

$$P(f) = \{n : \exists \text{ point of period}^* \ n \text{ for } f\}.$$

(*) $f^n(x) = x$ but $f^k(x) \neq x$ for $0 < k < n$

Theorem. Let $P \subset \mathbb{N}$ be any subset. There exists a continuous (non-differentiable) function f from the annulus into itself such that $P(f) = P$.

This function is constructed using a sequence of concentric circles and an extension to the annulus. The extension can also be made to a 1-dimensional continuum using a sequence of connecting arcs. Some of this answers an old question asked by W. Kuperberg in this seminar in 1972. [UHMPB #33]

[In previous book, page 60.]

148 February 27, 1989

Attendance: Kato, Tymchatyn, Lelek, Sly, M. Cook, Ingram, Feuerbacher

E. D. Tymchatyn presents his results on hereditarily locally connected spaces (h.l.c.). All spaces are assumed to be separable metric. These implications are known:

$$(\text{rim-finite}) \,\&\, (\text{connected}) \Rightarrow (\text{h.l.c.})$$
$$(\text{h.l.c.}) \,\&\, (\text{compact}) \Rightarrow (\text{rim-countable})$$

Each h.l.c. space is the union of a totally disconnected set and a countable set (Nishiura & Tymchatyn, 1976), but there exists an h.l.c. space which is <u>not</u> rim-countable (ibidem).

[non-compact trees]

<u>Theorem</u>. If X is a separable metric space, then X embeds as the set of end-points in a tree if and only if X has a basis $\{U_i\}_1^\infty$ of open sets such that, for $\operatorname{dist}(U_i, U_j) > 0$, X can be separated 'between U_i and U_j'.

Each totally disconnected set in an h.l.c. space satisfies the condition of that basis in the above theorem (joint with L. Oversteegen). A recent result of Mayer & Oversteegen is also used, namely, that each tree admits a convex metric. From it it follows that the set of end-points of a tree is of dimension ≤ 1.

<u>Corollary</u>. Each h.l.c. space has dimension ≤ 2.

The following questions remain open:

(1) Is each ~~suppose an~~ separable metric h.l.c. space of dimension ≤ 1?

(2) Can $\mathcal{H}(M)$ (the space of homeomorphisms of the Menger curve) embed as end-points of a tree?

(149) March 6, 1989

Attendance: H. Cook, Ingram, Kato, Lelek, Tymchatyn, M. Cook, Slye.

E. D. Tymchatyn presents his results (joint with Dębski) on accessible sets.

Theorem 1. Let X be a 2^{nd} category separable metric space, R an equivalence relation in X such that each R-equivalence class $R(x)$ is dense in X, and let $K \subset X$ be a compact set which intersects each R-equivalence class. Then there exists a non-empty closed set $L \subset K$ such that

$$cl\left(L \cap R(x)\right) = L \quad \text{for each } x \in X.$$

By this theorem, such a compact set K is not a cross-section, in a strong sense, generalizing a result of H. Cook.

Theorem 2. If X is a 2^{nd} category separable metric space, R is an equivalence relation in X with each R-equivalence class dense in X, then there does not exist a σ-compact cross-section

Let $X \subset S^2$ be a set, not dense in the sphere S^2. If R is an equivalence relation in X, we say that an R-equivalence class $R(x)$ is <u>external</u> provided there exists a continuum $L \subset S^2$ with

$$L \cap R(x) \neq \emptyset,$$
$$L \not\subset \mathrm{Cl}\, X,$$
$$L \cap R(y) = \emptyset \quad \text{for some } y \in X.$$

<u>Theorem 3.</u> Let $X \subset S^2$ be such that each non-empty open set in X is 2nd category. Let $R \subsetneqq X \times X$ be an equivalence relation such that each R-equivalence class $R(x)$ is 1st category, continuum-connected and dense in X. Then the union of all external R-equivalence classes is 1st category and F_σ in X.

[Cf. PAMS 1987,] for similar results.

Properties of S^2, like separation by unions of continua, are used in the proof of Thm. 3. The technique generalizes that of Krasinkiewicz whose approach to external components in indecomposable continua on the plane had motivated these investigations.

150 March 13, 1989

<u>Attendance:</u> Tymchatyn, Ingram, Lelek, M. Cook, Kato, Slye.

E. D. Tymchatyn continues his presentation of accessible sets. These composant-like decompositions have properties which thus far have been established only for composants of indecomposable continua.