A CHARACTERIZATION OF INDECOMPOSABLE COMPACT CONTINUA

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There are many characterizations of indecomposability of compact continua. There is wide interest in characterizing properties of a space in terms of open covers of the space. The purpose of this note is to characterize indecomposability of a compact continuum in terms of open covers of the continuum.

Space is assumed to be metric. If M is a compact continuum it is true that there is a sequence $G_1, G_2, G_3, \ldots$ of finite open covers of M such that (1) for each positive integer n, G_n is a coherent collection, (2) for each n, G_{n+1} is a strong refinement of G_n (the closure of each set in G_{n+1} is a subset of some set in G_n), (3) for each n, the mesh of G_n is less than $\frac{1}{n}$, (4) each set in G_n contains a point of M not in any other set in G_n and (5) M is the common part of $G_1^*, G_2^*, G_3^*, \ldots$. Such a sequence of finite open covers of M is called a <u>defining sequence for M</u>.

THEOREM. The compact continuum M is indecomposable if and only if there is a defining sequence $G_1, G_2, G_3, \ldots$ for M such that if i is a positive integer there is a positive integer j greater than i such that if G_j is the sum of two coherent collections L_1 and L_2 then L_1^* or L_2^* intersects every open set in G_i.

PROOF. Suppose M is a decomposable compact continuum and $G_1, G_2, G_3, \ldots$ is a defining sequence of M. Then M is the sum of two proper subcontinua

H and K. There is a point P in H which is not in K and a point Q in K which is not in H. Moreover, there is a positive integer i such that no set in G_i which contains P contains a point of K and no set in G_i which contains Q contains a point of H. Then if j is an integer greater than i and L_1 and L_2 are subcollections of G_j containing the sets in G_j which intersect H and K respectively, then (1) L_1 and L_2 are coherent subcollections of G_j, (2) G_j is the sum of L_1 and L_2, and (3) neither L_1^* nor L_2^* intersect every open set in G_i.

Suppose M is a compact continuum and it is true that if $G_1, G_2, G_3, \ldots$ is a defining sequence for M then there is a positive integer i such that if j is an integer greater than i then there exist coherent collections L_1 and L_2 whose sum is G_j and each of L_1^* and L_2^* fail to intersect some open set in G_i. Assume $i = 1$. Then there are two sets g and h in G_1 and a subsequence $G_{n_1}, G_{n_2}, G_{n_3}, \ldots$ of $G_1, G_2, G_3, \ldots$ such that $n_1 = 1$ and for each integer k greater than 1 G_{n_k} is the sum of two coherent collections L_1^k and L_2^k with the property that $(L_1^k)^*$ does not intersect g and $(L_2^k)^*$ does not intersect h. Then the sequential limiting sets H and K of the sequences $(L_1^2)^*, (L_1^3)^*, (L_1^4)^*, \ldots$ and $(L_2^2)^*, (L_2^3)^*, (L_2^4)^*, \ldots$, respectively, are two proper subcontinua of M whose sum is M.

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