TREE-LIKE CONTINUA AND SPAN

W.T. Ingram

1. Introduction

This paper is a continuation of a paper presented by the author at the Spring Topology Conference at Auburn University in 1976 [8]. At the present time the author knows of no question raised in that paper which has been settled. However, some recent results of the author lead to certain questions raised in this paper. Some other questions which were not raised there are raised here. Although there are relationships among questions from the various sections of the paper, we have divided the paper into sections of related topics. Throughout this paper the term continuum means compact, connected metric space, and the term mapping means continuous function. The span of a continuum M is defined to be $\sigma(M) = \text{l.u.b.}\{\epsilon \mid$ there exists a continuum Z in $M \times M$ such that $\pi_1(Z) = \pi_2(Z)$ and $\text{dis}(x, y) \geq \epsilon$ for all $(x, y) \in Z\}$. Here, π_1 and π_2 denote projection maps.

2. Homogeneity

In [11] we prove that each element of the collection H of [10] has positive span and that no continuum in the collection H is homogeneous. For a discussion of homogeneous continua, see the paper by Hagopian in this proceedings.

The argument that no member of H is homogeneous is a rather special argument designed for the members of H. However, it would be nice to know the answer to:

Question 1. Is there a homogeneous, tree-like continuum with positive span?

3. Hereditarily Equivalent Continua

In 1970 Cook [3] showed that each hereditarily equivalent continuum is tree-like. In that paper he proved the following theorem. If M is hereditarily equivalent and $\epsilon > 0$, there exists a homeomorphism h of M into M such that $d(x, h(x)) < \epsilon$ for each x in M.

This theorem may be of some use in settling:

Question 2. Is there an hereditarily equivalent continuum with positive span?

4. Plane Embeddings

The author has asked [8, Question 1] if there is an atriodic tree-like continuum which cannot be embedded in the plane. In a recent conversation with the author C. E. Burgess raised the following:

Question 3. (Burgess) Suppose M is an hereditarily indecomposable simple triod-like continuum such that every proper subcontinuum is a pseudo-arc (such a continuum which is not chainable, Burgess called a pseudo-triod). Can M be embedded in the plane?

Definition. The statement that the continuum M has property U means there exists an uncountable collection G of mutually exclusive continua in the plane such that each member of G is homeomorphic to M.

Question 4. Is there an atriodic tree-like continuum with positive span which has property U?

Laidacker [15] has shown that if C is a compactum such that every component of C is an atriodic tree-like continuum, then there is an atriodic tree-like continuum which contains C. If there is an atriodic tree-like continuum which does not have property U, then Laidacker's result would yield an answer to Question 1 of [8].

Recently, the author has constructed an hereditarily indecomposable tree-like continuum containing only degenerate subcontinua with span 0 [11]. It would be interesting to know if this continuum can be embedded in the plane.

5. Span Zero

Lelek [14] has asked if continua with span zero are chainable. The reader can see the importance of this question as it relates to several long-outstanding questions in the theory of continua from Questions 1, 2, and 4 of this paper.

If one proves that continua with span 0 are chainable, beside having a nice theorem one may provide a step toward classifying all the homogeneous plane continua.

If on the other hand one suspects that there are continua with span zero which are not chainable, perhaps solving one of the questions in this paper would provide a tool leading to the construction of such an example.

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