UNIVERSITY OF HOUSTON
MATHEMATICS PROBLEM BOOK (10/14/94)

Problem 1. *Suppose X is a continuum which cannot be embedded in any continuum Y such that Y is the union of a countable family of arcs. Does X contain a connected subset that is not path-connected? (E. D. Tymchatyn, 09/15/71)*

Comment: The answer is no. (E. D. Tymchatyn, 09/30/81)

Problem 2. *Suppose X is a continuum such that for each positive number ϵ there are at most finitely many pairwise disjoint connected sets in X of diameter greater than ϵ. Suppose, if Y is any continuous, monotonic, Hausdorff image of X, then Y can be embedded in a continuum Z which is the union of a countable family of arcs. Is every connected subset of X path-connected? (E. D. Tymchatyn, 09/15/71)*

Problem 3. *If X is a continuum such that every subcontinuum C of X contains a point that locally separates C in X, is X regular? (E. D. Tymchatyn, 09/15/71)*

Definition. A point p of a locally compact separable metric space L is a **local separating point** of L provided there exists an open set U of L containing p and two points x and y of the component containing p of U such that $U \setminus \{p\}$ is the sum of two mutually separated point sets, one containing x and the other containing y.

Definition. A continuum is **regular** if each two of its points can be separated by a finite point set.

Comment: The answer is no. (E. D. Tymchatyn, 03/19/79)

Problem 4. *If X is a Suslinean curve, does there exist a countable set A in X such that A intersects every nondegenerate subcontinuum of X? (A. Lelek, 09/22/71)*

Definition. A continuum is **Suslinean** if it does not contain uncountably many mutually exclusive nondegenerate subcontinua.

Comment: No. (P. Minc, 06/15/79)

Problem 5. *Can every hereditarily decomposable chainable continuum be embedded in the plane in such a way that each end point is accessible from its complement? (H. Cook, 09/29/71)*

Definition. A **simple chain** is a finite sequence $L_1, L_2, \cdots, L_n$ of open sets such that L_i intersects L_j if and only if $|i - j| \leq 1$. The terms of the sequence $L_1, L_2, \cdots, L_n$ are called the **links** of the chain. An **ϵ-chain** is a chain each of whose links has diameter less than ϵ. A continuum M is **chainable** if, for each positive number ϵ, M can be covered by an ϵ-chain.

Comment: No. (P. Minc and W. R. R. Transue, 02/27/90)

Problem 6. *Is it true that a chainable continuum can be embedded in the plane in such a way that every point is accessible from the complement if and only if it is Suslinean? (H. Cook, 09/29/71)*

Comment: That such a chainable continuum must be Suslinean follows from a result in *Bull. Polish Acad. Sci.* **9** (1960), 271–276. (A. Lelek, 09/29/71)

Comment: Yes. (P. Minc and W. R. R. Transue, 02/27/90)

Problem 7. *Is it true that the rim-type of any real (or half-ray) curve is at most 3? (A. Lelek, 09/29/71)*

Definition. A continuum is a **real curve** if it is a continuous 1-1 image of the real line and a **half-ray curve** if it is a continuous 1-1 image of the nonnegative reals.

Definition. A continuum is **rational** if each two of its points can be separated by a countable point set.

Definition. Suppose that X is a rational curve and a is the least ordinal for which there exists a basis $\mathcal{B}$ for X such that the a-th derivative of the boundary of each element of $\mathcal{B}$ is empty. Then a is the **rim-type** of X.

Comment: A positive solution follows from Nadler's work. (W. Kuperberg and P. Minc, 03/28/79)

Problem 8. *Does there exist a rational curve which topologically contains all real (or half-ray) curves? (A. Lelek, 09/29/71)*

Comment: Yes. (W. Kuperberg and P. Minc, 03/28/79)

Problem 9. *Does there exist a real curve which can be mapped onto each (weakly chainable) real curve? If not, does there exist a rational curve which can be mapped onto each (weakly chainable) real curve? (A. Lelek, 09/29/71)*

Definition. A continuum is **weakly chainable** if it is a continuous image of a chainable continuum.

Comment: Delete the parentheses, weakly chainable is needed. (W. Kuperberg and P. Minc, 03/28/79)

Problem 10. *Suppose D is a dendroid, S is a compact, finitely generated commutative semigroup of monotone surjections on D which has a fixed end point. Does S have another fixed point? (L. E. Ward, 10/20/71)*

Comment: Yes, even for λ-dendroids. (W. J. Gray and S. Williams, *Bull. Polish Acad. Sci.* **27** (1979), 599–604)

Problem 11. *Is each strongly Hurewicz space an absolute F_σ? (A. Lelek, 11/03/71)*

Comment: No. (E. K. van Douwen, 05/06/79)

Comment: The answer still seems to be unknown for the metric case. (A. Lelek, (08/31/94)

Definition. A regular space is a **Hurewicz space** if and only if, for each sequence of open coverings $Y_1, Y_2, \ldots,$ there exist finite subcollections $\mathcal{A}_i$ of Y_i such that $\mathcal{A}_1 \cup \mathcal{A}_2 \cup \cdots$ is a covering. A regular space X is **strongly Hurewicz** if and only if, for each sequence of open coverings $Y_1, Y_2, \cdots$ of X, there exist finite subcollections $\mathcal{A}_i$ of Y_i such that X is the union over i of the intersection over j of $\mathcal{A}^*_{i+j}$.

Notation. If G is a collection of sets, then G^* denotes the sum of all the sets in G.

Problem 12. *Is each product of two Hurewicz metric spaces a Hurewicz space? (A. Lelek, 11/03/71)*

Comment: No. (J. M. O'Farrell, 03/23/84)

Problem 13. *Is the product of two strongly Hurewicz metric spaces a strongly Hurewicz space? (H. Cook, 11/03/71)*

Problem 14. *Is every Lindelöf totally paracompact space Hurewicz? (D. W. Curtis, 11/10/71)*

Definition. A space X is **totally paracompact** if every basis for X has a locally finite subcollection covering X.

Problem 15. *Suppose f is an open mapping of a compact metric space X to the Sierpiński curve. Do there exist arbitrarily small closed neighborhoods U of x in X for which y is in Int $f(U)$ and $f|U$ is confluent? (A. Lelek, 11/24/71)*

Definition. The **Sierpiński curve** is a locally connected planar curve with infinitely many complementary domains whose boundaries are mutually exclusive simple closed curves.

Definition. A mapping f of a compact space X onto a compact space Y is **confluent** if, for each continuum C in Y, each component of $f^{-1}(C)$ is mapped onto C by f.

Comment: No. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 16. *Suppose f is a locally confluent and light mapping of a compact metric space X to the Sierpiński curve. Do there exist arbitrarily small closed neighborhoods U of x in X for which y is in Int $f(U)$ and $f|U$ is confluent? (A. Lelek, 11/24/71)*

Definition. A mapping f from a space X onto a space Y is **locally confluent** if, for each point y of Y, there is an open set U containing y such that $f^{-1}(\overline{U})$ is confluent.

Definition. A mapping is **light** if each point inverse is totally disconnected.

Comment: Yes. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 17. *Is there a space X on which the identity is strongly homotopically stable but there is an open mapping onto X which is not strongly homotopically stable? (H. Cook, 11/24/71)*

Definition. Suppose f is a mapping of a metric space X into a metric space Y and a is a point of X. Then f is

(1) **unstable** at a provided, for each $\epsilon > 0$, there exists a mapping g from X into Y such that $d\left(f\left(x\right), g\left(x\right)\right) < \epsilon$ for x in X and $f(a)$ in $g(X)$;
(2) **homotopically unstable** at a provided, for each $\epsilon > 0$, there exists a homotopy h of $X \times [0,1]$ into Y such that if x is in X and t is in $I = [0,1]$, then

$$h(x,0) = f(x), \qquad d\big(h(x,0), h(x,t)\big) < \epsilon, \quad \text{and} \qquad f(a) \neq h(x,1);$$

(3) **strongly homotopically stable** at a provided for each homotopy h of $X \times I$ into Y, where $h(x,0) = f(x)$, we have $h(a,t) = f(a)$ for t in I.

Problem 18. *Is it true that each open mapping of a continuum X onto the Sierpiński curve (or the Menger curve) is strongly homotopically stable at each point of X? (A. Lelek, 11/24/71)*

Comment: No. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 19. *Does each strictly non-mutually aposyndetic continuum with no weak cut point contain uncountably many mutually exclusive triods? (H. Cook, 12/08/71)*

Definition. A continuum M is **mutually aposyndetic** if, for each two points A and B of M, there exist mutually exclusive subcontinua H and K of M containing A and B, respectively, in their interiors. A continuum is **strictly non-mutually aposyndetic** if each two of its subcontinua with interiors intersect.

Definition. A point x of a continuum M is a **weak cut point** of M if there are two points p and q of M such that every subcontinuum of M that contains both p and q also contains x.

Comment: E. E. Grace has shown that every planar strictly non-mutually aposyndetic continuum has a weak cut point.

Problem 20. *Suppose f is an open continuous mapping of a continuum X onto Y. Does there exist a space X^* such that X is a subset of X^*, X^* is a locally connected continuum, and there is an extension f^* of f from X^* to Y^* such that f^* is open and $f^*(X)$ does not intersect $f^*(X^* \setminus X)$? Can X^* be the Hilbert cube? If X is a curve, can X^* be the Menger universal curve? (A. Lelek, 12/08/71)*

Comment: The answer to the first and the last question is yes. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 21. *If X is a planar continuum which has only a finite number of complementary domains and X has property A then is X connected im kleinen? (A. Lelek, 01/26/72)*

Definition. A connected space X is **aposyndetic** at H with respect to K if there is a closed connected subset of X with H in its interior and not intersecting K, and X is **aposyndetic** if it is aposyndetic at each point with respect to every other point.

Definition. A space is **connected im kleinen** if it is aposyndetic at each point with respect to each closed point set not containing that point.

Definition. Suppose M is a connected metric space and x and y are two points of M. If, for each connected and closed subset of K of $M - \{x\}$ containing y, M is aposyndetic at x with respect to K, then M is said to have **property** A at x with respect to y. A continuum has **property** A if it has property A at each point with respect to every other point.

Comment: Yes, quite general solution. (T. Maćkowiak and E. D. Tymchatyn, 09/30/81)

Problem 22. *Does every totally non-semi-locally connected continuum contain a dense G_δ set of weak cut points? (E. E. Grace, 02/02/72)*

Definition. A continuum M is **semi-locally connected** at the point p if, for every domain D containing p, there exits a domain E lying in D and containing p such that $M \setminus E$ has only a finite number of components. A continuum is **totally non-semi-locally connected** if it is not semi-locally connected at any point.

Problem 23. *Does every totally non-semi-locally connected bicompact Hausdorff continuum contain a weak cut point? (E. E. Grace, 02/02/72)*

Problem 24. *Is it true that if X is a simple tree and $w(X) > \epsilon > 0$, then there is a simple triod T in X such that $w(T) > \epsilon$? (A. Lelek, 02/09/72)*

Definition. For any compact metric space X, the **width** $w(X)$ of X is the l.u.b. of the set of all real numbers a which satisfy the following condition: for each $\epsilon > 0$, there exists a finite open cover $\mathcal{C}$ of X such that $\mathrm{mesh}(\mathcal{C}) < \epsilon$ and for each chain $\mathcal{C}'$ which is a subcollection of $\mathcal{C}$ there is a member A of $\mathcal{C}$ such that $d(A, \mathcal{C}'^*) \geq a$.

Comment: No. (F. O. McDonald, 06/01/74)

Problem 25. *Suppose X is a tree-like continuum, $w(X) > 0$, and f is a continuous mapping from X to a chainable continuum Y. Is it true that there exist, for each a such that $0 < a < w(X)$, two points x and y such that $d(x, y) = a$ and $f(x) = f(y)$? (A. Lelek, 02/09/72)*

Definition. If X is a metric space, a mapping f from X to a space Y is an **ϵ-map** if, for each point y of Y, $\operatorname{diam}\bigl(f^{-1}(y)\bigr) \leq \epsilon$. If $\mathcal{C}$ is a collection of continua, a continuum M is **$\mathcal{C}$-like** if, for every positive number ϵ, there exists an ϵ-map of M onto an element of $\mathcal{C}$. In particular, a continuum is **tree-like** if, for some collection $\mathcal{C}$ of trees, M is $\mathcal{C}$-like.

Problem 26. *Let M be a locally compact metric space and a be a point of M. Then a is unstable if and only if there exists a homotopy $f(\cdot, t)$ of M to M such that $f(\cdot, 0)$ is the identity and a is in $f(M, t)$ for each $t > 0$. Also, if and only if there exists a retraction r of $M \times [0, 1]$ to $M \times \{0\}$ such that $r^{-1}\bigl((a, 0)\bigr) = \{\,(a, 0)\,\}$. Is local compactness necessary in this theorem? (W. Kuperberg, 09/13/72)*

Problem 27. *Suppose M is an n-dimensional compact metric space and A is the set of all unstable points of M. Does there exist a homotopy $f(\cdot, t)$ of M to M such that $f(\cdot, 0)$ is the identity and $f(M, t)$ and A do not intersect for each $t > 0$? (W. Kuperberg, 09/13/72)*

Comment: No. (W. Kuperberg and P. Minc, 03/28/79)

Problem 28. *Prove that the $\sin(1/x)$ curve is not pseudo-contractible. (W. Kuperberg, 09/20/72)*

Comment: Done. (W. Debski, 05/08/90)

Definition. Let C be a continuum and a and b be two points of C. Suppose f and g are two mappings from a space X into a space Y. Then f and g are **(C, a, b)-homotopic** provided there exists a mapping F of $X \times C$ into Y such that $F(x, a) = f(x)$ and $F(x, b) = g(x)$, for each x in X. We say that f is **pseudo-homotopic** to g provided there exist C, a, and b such that f and g are (C, a, b)-homotopic. A space is **pseudo-contractible** provided the identity mapping is pseudo-homotopic to a constant.

Problem 29. *Does there exist a 1-dimensional continuum which is pseudo-contractible but is not contractible? (W. Kuperberg, 09/20/72)*

Problem 30. *Suppose X and Y are continua (or even polyhedra), x is in X, y is in Y, and there exist neighborhoods U and V of x and y in X and Y, respectively, such that a homeomorphism of U onto V transforms x onto y. Is it true that if x is pseudo-unstable in X, then y is pseudo-unstable in Y? (W. Kuperberg, 09/20/72)*

Definition. A point of M is **C_a-unstable** (where a is in C, C a continuum) provided there exists a mapping F of $M \times C$ into M such that

(1) $F(x, a) = x$ for x in M, and
(2) $F(x, c) \neq y$ for x in M, c in C, and $a \neq c$.

We say that y is **pseudo-unstable** provided there exists a continuum C such that y is C_a-unstable.

Problem 31. *Is it true that the pseudo-arc is not pseudo-contractible? (W. Kuperberg, 09/20/72)*

Problem 32. *Does there exist a mapping f of the circle or the plane onto itself such that f^n has a fixed point for each $n \geq 2$, but f does not have a fixed point? (W. Kuperberg, 10/11/72)*

Comment: Yes; L. Block, *Trans. Amer. Math. Soc.* **260** (1980), 553–562, and, independently, M. Cook, oral communication, 1988. (W. T. Ingram, 06/30/89)

Problem 33. *Suppose P is a subset of the positive integers with the property that, if k is in P, then $n \cdot k$ is in P for each positive integer n. Does there exist a locally connected continuum X and a mapping f of X onto X such that f^n has a fixed point if and only if n is in P? (W. Kuperberg, 10/11/72)*

Comment: Yes. (M. Cook, 02/20/89)

Problem 34. *Is it true that if f is a mapping of a tree-like continuum into itself, then there exits an n such that f^n has a fixed point? (W. T. Ingram, 10/11/72)*

Comment: No. (P. Minc, 06/15/91)

Problem 35. *Suppose f is a continuous mapping of a continuum X onto a continuum Y, $Y = H \cup K$ is a decomposition of Y into subcontinua H and K, $f|f^{-1}(H)$ and $f|f^{-1}(K)$ are confluent, and $H \cap K$ is a continuum which does not cut Y and is an end continuum of both H and K. Is f confluent? (W. T. Ingram, 10/11/72)*

Problem 36. *Suppose f is a weakly confluent and locally confluent mapping of a continuum X onto a tree-like continuum Y. Does there exist a subcontinuum L of X such that $f(L) = Y$ and $f|L$ is confluent? (D. R. Read, 10/25/72)*

Comment: No. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 37. *Suppose $(X, \leq)$ is a finite partially ordered set. By taking $\overline{\{y\}} = \{\, x \mid x \leq y \,\}$ one gets a T_0 topology on X; $\overline{A}$ is the union of the closures of the points of A, and then a mapping f of X into X is continuous if and only if f is order preserving. A retraction r of X onto Y is* **relating** *provided that x in $X \setminus Y$ implies x and $r(x)$ are related. Does there exist a unique minimal relating retract of X? (L. E. Ward, 11/29/72)*

Problem 38. *Suppose X in the previous problem is $A \cup B$, where A and B are closed sets and A, B, and $A \cap B$ have the fixed point property. Does X have the fixed point property? (L. E. Ward, 11/29/72)*

Problem 39. *Suppose X is a finite partially ordered set with the fixed point property. Is X unicoherent? (L. E. Ward, 11/29/72)*

Problem 40. *Does there exist a nonseparating (locally compact) subset of the plane which is neither compact nor open in the plane such that if f is a continuous and reversible function of X onto a subset of the plane, then f is a homeomorphism? (A. Lelek, 02/07/73)*

Problem 41. *Suppose that the plane is the union of two disjoint sets A and B neither of which contains a Cantor set. Is each continuous and 1-1 function of A into the plane a homeomorphism? (A. Lelek, 02/07/73)*

Comment: No. (E. K. van Douwen, 07/14/80)

Problem 42. *Suppose X is a compact metric space and S is the set of all stable points of X. Suppose* $\dim X$ *is finite and a is an infinite cycle in S. Is it true that if $a \sim 0$ in X, then $a \sim 0$ in S? (T. Ganea, 03/07/73)*

Comment: Yes, solved by Namioka. (W. Kuperberg and P. Minc, 03/28/79)

Problem 43. *Suppose X is a curve is contractible, or pseudo-contractible, or contractible to a proper subset only. Does there exist in X a point which is unstable (or pseudo-unstable, respectively, or otherwise)? (W. Kuperberg, 03/07/73)*

Problem 44. *Suppose X is a metric space; does 'superstable' in X imply 'stable'? (W. Kuperberg, 03/07/73)*

Definition. Given a point p of a space X, p is **superstable** if it is not pseudo-unstable.

Problem 45. *Is it true that if Y is an arcwise connected continuum such that each continuous mapping of a continuum onto Y is weakly confluent, then Y is an arc? (A. Lelek, 03/21/73)*

Comment: Yes, since Y is irreducible, unicoherent, and not a triod. (J. Grispolakis, 03/19/79)

Problem 46. *Suppose Y is a compact metric space such that the Borsuk modified fundamental group of Y is 0 and f is a local homeomorphism of X onto Y. Must f be a homeomorphism? (W. Kuperberg, 04/11/73)*

Problem 47. *Do atomic mappings preserve semi-aposyndesis (or mutual aposyndesis) of continua? (W. T. Ingram, 04/18/73)*

Definition. If f is a mapping of a continuum X onto a continuum Y and, for each subcontinuum K of X such that $f(K)$ is nondegenerate $K = f^{-1}(f(K))$, then f is **atomic**. Atomic maps are monotone.

Comment: Yes, such mappings must be either homeomorphisms or constant mappings. (T. Maćkowiak, 09/30/81)

Problem 48. *Is each unicoherent and mutually aposyndetic continuum locally connected? (L. Gibson, 04/18/73)*

Comment: No, but the answer is unknown for such continua which are 1-dimensional. (T. Maćkowiak, 09/30/81)

Problem 49. *Suppose f is an open mapping of a continuum X onto Y and y is a branch-point of Y. Is there a branch-point x of X such that $f(x) = y$? (V. Parr, 04/25/73)*

Definition. A **branch point** of a continuum is the vertex of a simple triod lying in that continuum.

Comment: No, but the answer is yes if f is light. (T. Maćkowiak, 09/30/81)

Problem 50. *Suppose f is a weakly confluent mapping of a continuum X onto Y and y is a branch-point of Y. Is there a branch-point x of X such that $f(x) = y$? (V. Parr, 04/25/73)*

Comment: No. (T. Maćkowiak, 09/30/81)

Problem 51. *Is it true that each polyhedron is the weakly confluent image of a polyhedron whose fundamental group is trivial?*

Comment: False, T2. (J. Grispolakis, 03/19/79)

Problem 52. *Are strongly regular curves inverse limits of connected graphs with monotone simplicial bonding maps? (B. B. Epps, 09/19/73)*

Comment: No. (E. D. Tymchatyn, 10/27/74)

Problem 53. *If X is the inverse limit of connected graphs with monotone simplicial retractions as bonding maps, is X the weakly confluent image of a dendrite? (B. B. Epps, 09/19/73)*

Comment: Yes. (J. Grispolakis and E. D. Tymchatyn, 03/19/79)

Problem 54. *Is it true that X is the inverse limit of connected graphs with monotone simplicial retractions as bonding maps if and only if X is locally connected and each cyclic element of X is a graph? (B. B. Epps, 09/19/73)*

Definition. If M is a semi-locally connected continuum, a **cyclic element** of M is either a cut point of M, an end point of M, or a nondegenerate subset of M which is maximal with respect to being a connected subset of M without cut points.

Problem 55. *Does each continuous mapping of a continuum onto a chainable continuum have property F? (A. Lelek, 09/26/73)*

Definition. A mapping f of a continuum X onto a continuum Y has **property** F provided that there exists a point x in X such that if K is a subcontinuum of Y containing $f(x)$ and C is the component of $f^{-1}(K)$ containing x, then $f(C) = K$.

Comment: No. (J. B. Fugate, 06/16/79)

Problem 56. *Does each (weakly confluent mapping of a continuum onto an irreducible continuum have property F? (A. Lelek, 09/26/73)*

Comment: No. (J. B. Fugate, 06/16/79)

Problem 57. *Is it true that each uniformly pathwise connected continuum is uniformly arcwise connected? (W. Kuperberg, 09/26/73)*

Definition. Suppose X is a metric space. A **path** is a continuous mapping of $[0, 1]$ into X. A collection P of paths in X is **uniform** provided, for each $\epsilon > 0$, there exists a positive integer n such that, for each p in P, there are points

$$0 = t_0 < t_1 < t_2 < \cdots < t_n = 1$$

such that $\operatorname{diam}([t_{i-1}, t_i]) \leq \epsilon$ for $i = 1, \cdots, n$. We say X is **uniformly pathwise connected** provided that, for each two points a and b of X, there exists a uniform collection P of paths in X joining a and b such that the union of their images is X. We say that X is **uniformly arcwise connected** provided that, for each two points a and b of X, there exists a uniform collection P of homeomorphisms of $[0, 1]$ into X joining a and b such that the union of their images is the union of all arcs in X joining a and b.

Problem 58. *Suppose f is a continuous mapping of a chainable continuum X onto a nonchainable continuum Y. Does there exist a subcontinuum X' of X mapped onto Y under f such that $f|X'$ is not weakly confluent? (A. Lelek, 10/03/73)*

Comment: No. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 59. *Does a continuum with zero surjective span have zero span? (A. Lelek, 11/07/73)*

Definition. Let f be a mapping from a (connected if necessary) space X to a metric space Y and let S be the least upper bound of numbers a such that there exists a connected set C in $X \times X$ both of whose projections are the same point set and, for each point (x, y) in C, $d(f(x), f(y)) \geq a$. Then S is the **span** of f. If we require that both projections of C be all of X, then S is the **surjective span** of f. If we require only that one projection of C be a subset of the other, then S is the **semi-span** of f; and if we require that one projection of C be all of X, then S is the **surjective semi-span** of f. If X is a (connected) metric space, then the **span**, **surjective span**, **semi-span**, and **surjective semi-span** of X are the span, surjective span, semi-span, and surjective semi-span, respectively, of the identity map of X onto itself.

Notation. If Z is a (connected) metric space or a mapping from a (connected) metric space to a metric space, then

(1) the span of Z is denoted by $\sigma(Z)$,
(2) the surjective span of Z by $\sigma^*(Z)$,
(3) the semi-span of Z by $\sigma_0(Z)$, and
(4) the surjective semi-span of Z by $\sigma_0^*(Z)$.

Problem 60. *Is it true that if $Y = G_1 \cup G_2$ where G_1 and G_2 are open and $f|f^{-1}(\overline{G_i})$ is an MO-mapping for $i = 1, 2$, then f is an MO-mapping? (A. Lelek, 11/21/73)*

Definition. An **MO-mapping** is a composition gf where g is monotone and f is open.

Comment: Spaces are compact metric. (A. Lelek, 11/21/73)

Problem 61. *Suppose $\mathcal{E}$ is the class of all finite one point unions of circles. Is it true that if X and Y are shape-irreducible continua possessing the same shape and there exist curves C and C' in $\mathcal{E}$ such that X is C-like and Y is C'-like, then there exists a C'' in $\mathcal{E}$ such that both X and Y are C''-like? (A. Lelek, 12/05/73)*

Definition. A continuum X is said to be **shape-irreducible** if and only if no proper subcontinuum of X is of the same shape as X.

Comment: Trivially yes, but there even exists a shape-irreducible figure-eight-like continuum with the shape of a circle that is not circle-like. (J. Segal and S. Spiez, 05/07/86)

Problem 62. *Are all shapes of polyhedra stable? (W. Kuperberg, 04/03/74)*

Problem 63. *Given a polyhedron P. Does there exist a polyhedron Q of the same shape as P and shape stable? (W. Kuperberg, 04/03/74)*

Problem 64. *Is each uniformly path-connected continuum g-contractible? (D. Bellamy, 04/03/74)*

Definition. A space X is g-contractible provided there exists a mapping f of X onto X such that f is homotopic to a constant mapping.

Problem 65. *Is it true that, for each uniformly path-connected continuum X, there exists a compact metric space Y and two mapping f, from X onto the cone over Y, and g, from the cone over Y onto X? (D. Bellamy, 04/03/74)*

Comment: No. (J. Prajs, 03/01/89)

Problem 66. *If C is a connected Borel subset of a finitely Suslinean continuum X, is C arcwise connected? (E. D. Tymchatyn, 09/26/74)*

Definition. A continuum X is **finitely Suslinean** if, for each positive number ϵ, X does not contain infinitely many mutually exclusive subcontinua of diameter greater than ϵ.

Definition. The family $\mathcal{F}$ of **Borel sets** of a space S is the smallest family satisfying the conditions:

1. Every closed set belongs to $\mathcal{F}$;
2. If X is in $\mathcal{F}$, then $S \setminus X$ is in $\mathcal{F}$; and
3. The countable intersection of elements of $\mathcal{F}$ belongs to $\mathcal{F}$.

Comment: Yes, for subsets of regular curves. (D. H. Fremlin, 04/12/90)

Problem 67. *Suppose X is a continuum such that, if C is a subcontinuum of X then the set of all local separating points of C is not the union of countably many closed disjoint proper subsets of C. Then, is every connected subset of X arcwise connected? (E. D. Tymchatyn, 09/26/74)*

Problem 68. *If for each subcontinuum C of the continuum X, the set of all local separating points of C is connected, then is every connected subset of X arcwise connected? (E. D. Tymchatyn, 10/03/74)*

Problem 69. *Do hereditarily indecomposable tree-like continua have the fixed point property? (B. Knaster, 11/21/74)*

Problem 70. *Do uniquely λ-connected (or uniquely δ-connected) continua have the fixed point property for homeomorphisms? (A. Lelek, 11/21/74)*

Definition. A continuum is **δ-connected** if each two of its points can be connected by an hereditarily decomposable irreducible subcontinuum. A continuum is **λ-connected** if each two of its points p and q can be joined by an irreducible continuum of type λ, i.e. an irreducible continuum from p to q which has arcatomic subsets, no one of which has interior.

Problem 71. *Suppose that X is a continuum such that, if $\epsilon > 0$, $C_1, C_2, \cdots$ are mutually separated connected sets of diameter greater than ϵ, and $C_1 \cup C_2 \cup \cdots$ is connected, then there exist mutually exclusive arcs $A_1, A_2, \cdots$ such that, for some subsequence $D_1, D_2, \cdots$ of $C_1, C_2, \cdots$ and each i, A_i is a subset of D_i, and $\operatorname{diam}(A_i) > \epsilon$. Is X locally connected? (E. D. Tymchatyn, 01/27/75)*

Problem 72. *Suppose X is a continuum such that each σ-connected subset of X is a semi-continuum. Is X locally connected? (J. Grispolakis, 01/27/75)*

Definition. A **semi-continuum** is a continuumwise connected point set.

Problem 73. *Suppose X is a continuum such that each σ-connected F_σ subset of X is a semi-continuum. Is X semi-aposyndetic? (J. Grispolakis, 01/27/75)*

Definition. A space X is **semi-aposyndetic** if, for each two of its points, X is aposyndetic at one of them with respect to the other.

Problem 74. *Let f be a perfect pseudo-confluent mapping of an hereditarily normal, hereditarily locally connected and hereditarily σ-connected space onto a complete metric space Y. Is Y hereditarily σ-connected? (J. Grispolakis, 02/03/75)*

Definition. A mapping from X to Y is **pseudo-confluent** if every irreducible continuum in Y is the image of a continuum in X.

Comment: The answer is yes if X is also a $(Q = C)$-space; i.e., if the quasi-components of any subset of X are its components. (J. Grispolakis, 02/03/75)

Problem 75. *Suppose X is a separable metric space which possesses an open basis $\mathcal{B}$ such that, the set $X \setminus G$ is the union of a collection of closed open subsets of X. Is X embeddable in an hereditarily locally connected space? (E. D. Tymchatyn, 03/03/75)*

Comment: Yes. (L. G. Oversteegen and E. D. Tymchatyn, 08/18/92)

Problem 76. *Suppose X is a separable metric space such that, if A and B are connected subsets of X, then $A \cap B$ is connected. Is it true that if C is a set of non-cut points of X, then $X \setminus C$ is connected? (L. E. Ward, 03/03/75)*

Problem 77. *Is it true that a continuum X is regular if and only if every infinite sequence of mutually disjoint connected subsets of X is a null sequence? (E. D. Tymchatyn, 03/03/75)*

Problem 78. *Is it true that, for each decomposition of a finitely Suslinean continuum into countably many disjoint connected sets, at least one of them must be rim-compact? (T. Nishiura, 03/03/75)*

Definition. A space is said to be **locally peripherally compact**, or **rim compact**, at p if every open set containing p contains an open set containing p and having a compact boundary.

Comment: No. (E. D. Tymchatyn, 09/30/81)

Problem 79. *Is it true that no biconnected set with a dispersion point can be embedded into a rational continuum? (J. Grispolakis, 03/03/75)*

Definition. A point p of a nondegenerate connected space X is an **explosion point**, or **dispersion point**, of X provided that $X \setminus \{p\}$ is totally disconnected.

Definition. A space is **biconnected** if it is not the sum of two mutually exclusive nondegenerate connected point sets.

Problem 80. *Is it true that a separable metric space is embeddable into a rational continuum if and only if it possesses an open basis whose elements have scattered boundaries? (E. D. Tymchatyn, 03/03/75)*

Definition. A point set X is **scattered** if every subset Y of X has a point that is not a limit point of Y.

Comment: Yes. (E. D. Tymchatyn, 09/30/81)

Problem 81. *Is it true that each continuum of span zero is chainable? (H. Cook and A. Lelek, 05/15/75)*

Problem 82. *Is it true that each continuum of surjective semi-span zero is arc-like? (A. Lelek, 05/15/75)*

Problem 83. *Suppose X is a connected metric space.*

(1) *Is the span of X less than or equal to twice the surjective span of X?*
(2) *Is the semi-span of X less than or equal to twice the surjective semi-span of X?*
(3) *Is the surjective semi-span of X less than or equal to twice the surjective span of X?*
(4) *If T is a simple triod, is the surjective span of T equal to the surjective semi-span of T?*

(A. Lelek, 05/15/75)

Problem 84. *Is the confluent image of an arc-like continuum arc-like? (A. Lelek, 10/21/75)*

Problem 85. *If f is a confluent mapping of an acyclic (or tree-like or arc-like) continuum onto a continuum Y, is $f \times f$ confluent? (A. Lelek, 10/21/75)*

Problem 86. *Do confluent maps of continua preserve span zero? (H. Cook and A. Lelek, 10/21/75)*

Problem 87. *Is every regularly submetrizable Moore space completely regular? (H. Cook, 10/13/76)*

Definition. If (X, T_1) is a topological space and there exists a subcollection T_2 of T_1 such that (X, T_2) is metrizable, then (X, T_1) is **submetrizable**. If, in addition, for each point p and open set O in T_1 containing p, there exists an open set D in T_1 containing p whose closure with respect to T_2 is a subset of O, then (X, T_1) is **regularly submetrizable**.

Problem 88. *Is every homogeneous continuum bihomogeneous? (B. Fitzpatrick, 10/27/76)*

Definition. Suppose X is a space such that, for each two points a and b, there is a homeomorphism of X onto itself such that $h(a) = b$ [and $h(b) = a$]. Then X is **homogeneous** [respectively, **bihomogeneous**].

Comment: No. (K. Kuperberg, 02/02/88)

Problem 89. *Does there exist a noncombinatorial triangulation of the* 4*-sphere? (C. E. Burgess, 10/12/77)*

Problem 90. *Is there an hereditarily equivalent continuum other than an arc or a pseudo-arc? (H. Cook, 11/02/77)*

Definition. A continuum is **hereditarily equivalent** if it is homeomorphic to each of its nondegenerate subcontinua.

Problem 91. *Do hereditarily equivalent continua have span zero? (H. Cook, 11/02/77)*

Problem 92. *If M is a continuum with positive span such that each of its proper subcontinua has span zero, does every nondegenerate monotone continuous image of M have positive span? (H. Cook, 11/02/77)*

Comment: No. (J. F. Davis and W. T. Ingram, 04/30/86; *Fund. Math.* **131** (1988), 13–24)

Problem 93. *Does there exist a homogeneous tree-like continuum of positive span? (W. T. Ingram, 02/08/78)*

Comment: Not in the plane. (L. G. Oversteegen and E. D. Tymchatyn, 07/23/80)

Problem 94. *If M is a plane continuum with no weak cut point, is M λ-connected? (C. L. Hagopian, 04/12/78)*

Comment: Yes, but there exists a counterexample in the 3-dimensional Euclidean space. (C. L. Hagopian, 04/01/79)

Problem 95. *Is the countable product of λ-connected continua λ-connected? (C. L. Hagopian, 04/12/78)*

Comment: Yes, but it is unknown if the product of any two continua is λ-connected. (C. L. Hagopian, 04/07/86)

Problem 96. *Is the image under a local homeomorphism of a δ-connected continuum also δ-connected? (C. L. Hagopian, 04/12/78)*

Problem 97. *Can it be proven, without extraordinary logical assumptions, that the plane is not the sum of fewer than $\mathfrak{c}$ mutually exclusive continua? (H. Cook, 04/19/78)*

Comment: The plane is not the sum of fewer than $\mathfrak{c}$ mutually exclusive continua each of which is either Suslinean or locally connected. (H. Cook, 04/19/78)

Problem 98. *Is it true that if (X, T_1) is normal, (X, T_2) is compact, T_2 is a subcollection of T_1, $\operatorname{ind}(X, T_1) = 0$, and $\operatorname{ind}(X, T_2) > 0$, then (X, T_1) fails to be a Hurewicz space? (A. Lelek, 09/13/78)*

Problem 99. *Is the Sorgenfrey line totally paracompact? (A. Lelek, 09/13/78)*

Comment: No. (J. M. O'Farrell, 09/01/80)

Problem 100. *Suppose f is a light open mapping from a continuum M onto a continuum N, B is a smooth dendroid lying in N, and x is a point of $f^{-1}(B)$. Does there exist a smooth dendroid A in M such that x is in A and $f|A$ is a homeomorphism? (J. B. Fugate, 10/18/78)*

Definition. A dendroid X is **smooth** if there is a point p in X such that, if $x_1, x_2, \cdots$ is a sequence of points of X converging to the point x, then the superior limit of the arcs $[p, x_n]$ is the arc $[p, x]$.

Comment: Example 2 of *Colloq. Math.* **38** (1978), 193–196, gives a negative solution. (T. Maćkowiak, 01/25/81)

Problem 101. *Suppose M is a continuum and f is a monotone mapping of M onto $[0,1]$ such that, if D is a closed proper subset of M which is mapped onto $[0,1]$ by f, then $f|D$ is not monotone. Is M irreducible? (L. Mohler and L. E. Ward, 10/18/78)*

Problem 102. *Is there a non-locally connected continuum M such that there exists a retraction of 2^M to $C(M)$? If M is the cone over a compact set with only one limit point, is $C(M)$ a retract of 2^M? (J. B. Fugate, 10/18/78)*

Problem 103. *Suppose M is an hereditarily unicoherent and hereditarily decomposable continuum and f is a continuous mapping M into 2^M. Is there a point x of M such that x is in $f(x)$? (J. B. Fugate, 10/18/78)*

Problem 104. *Is the open image of a circle-like continuum always circle-like or arc-like? (J. B. Fugate, 10/18/78)*

Problem 105. *Suppose M is an atriodic 1-dimensional continuum and G is an upper semi-continuous collection of continua filling up M such that M/G and every element of G are chainable. Is M chainable? (H. Cook and J. B. Fugate, 10/18/78)*

Comment: It follows from a result of Sher that, even if M contains a triod, if M/G and every element of G are tree-like, then M is tree-like. (H. Cook, 10/18/78)

Comment: Yes if the requirement that M is 1-dimensional is replaced by M is strongly unicoherent and M/G is hereditarily decomposable. (W. Dwayne Collins, 03/15/82)

Definition. A continuum X is **strongly unicoherent** provided X is unicoherent and each proper subcontinuum with interior is unicoherent.

Comment: Yes if M is hereditarily indecomposable and both M/G and every element of G are pseudo-arcs. (W. Lewis, 05/10/82)

Comment: No. (J. F. Davis and W. T. Ingram, 04/30/86; *Fund. Math.* **131** (1988), 13–24)

Problem 106. *If M is tree-like and every proper subcontinuum of M is chainable, is M almost chainable? (J. B. Fugate, 10/18/78)*

Definition. A continuum M is **almost chainable** if, for each positive number ϵ, there exists an open cover D of M with mesh ϵ and a chain C of elements of D and an end link L of C such that no element of $D \setminus C$ intersects any link of C other than L and every point of M is at a distance less than ϵ from some link of C.

Problem 107. *Suppose $M_1, M_2, \cdots$ is a sequence of mutually disjoint continua in the plane converging to the continuum M homeomorphically. Is M circle-like or chainable? (J. B. Fugate, 10/18/78)*

Definition. The statement that the sequence $M_1, M_2, \cdots$ **converges homeomorphically to the continuum** M means there exists a sequence $h_1, h_2, \cdots$ of homeomorphisms such that, for each positive integer i, h_i is a homeomorphism from M_i onto M and for each positive number ϵ there exists a positive integer N such that if $j > N$ then, for all x, $\operatorname{dist}(h_j(x), x) < \epsilon$.

Problem 108. *If M is a uniquely arcwise connected continuum, does each light open mapping of M onto itself have a fixed point? (J. B. Fugate and B. McLean, 10/18/78)*

Comment: No. (L. G. Oversteegen, 01/22/80)

Problem 109. *Do pointwise periodic homeomorphisms on tree-like continua have a fixed point? (J. B. Fugate and B. McLean, 10/18/78)*

Problem 110. *Do disk-like continua have the fixed point property for periodic homeomorphisms? (J. B. Fugate and B. McLean, 10/18/78)*

Problem 111. *If M is a tree-like continuum with the fixed point property, does $M \times [0,1]$ have the fixed point property? (J. B. Fugate, 10/18/78)*

Problem 112. *If M is a contractible continuum, do periodic homeomorphisms on M have fixed points? (J. B. Fugate, 10/18/78)*

Problem 113. *Suppose M is a continuum which is not the sum of a countable monotonic collection of proper subcontinua. Is M irreducible about some finite set? (J. B. Fugate, 10/18/78)*

Problem 114. *Is it true that, if X is a compact metric space, P is a 1-dimensional connected polyhedron with a geodesic metric and f is an essential mapping of X into P, then the span of f is greater than or equal to the span of P? (A. Lelek, 12/06/78)*

Comment: No, but a similar and quite strong result in this direction has been recently established by H. Kato, A. Koyama, and E. D. Tymchatyn. (A. Lelek, 05/15/89)

Problem 115. *Does every atriodic 2-dimensional continuum contain a 2-dimensional indecomposable continuum?*
(H. Cook, 02/19/79)

Problem 116. *If S is a compact, uniquely divisible, topological semigroup with 0, 1, and no other idempotents, must S have non-zero cancellation? (D. R. Brown, 03/02/79)*

Problem 117. *Suppose that M is a tree-like continuum such that, for each two points A and B of M, the diagonal in $M \times M$ intersects every continuum containing both (A, B) and (B, A). Does M have span zero? (H. Cook, 03/02/79)*

Problem 118. *Suppose H and K are two continua with span zero whose intersection is connected and whose sum is atriodic. Does their sum have span zero? (E. Duda, 03/02/79)*

Problem 119. *Suppose Y is a nondegenerate locally connected continuum and each cyclic element of Y is a completely regular continuum. Is it true that there exists a continuum X in the plane and a monotone open map of X onto Y such that $f^{-1}(y)$ is a (nondegenerate) decomposable continuum for each y in Y? (J. Krasinkiewicz, 03/07/79)*

Definition. A continuum is **completely regular** if each nondegenerate subcontinuum has a nonempty interior.

Comment: The answer is yes if Y itself is a completely regular continuum in the plane. (T. Maćkowiak and E. D. Tymchatyn, 01/25/81)

Problem 120. *Suppose M is the 'canonical' Knaster indecomposable continuum (obtainable by identifying each point of the dyadic solenoid with its inverse) and h is a homeomorphism of M onto itself. Does h leave two points fixed? (W. S. Mahavier, 03/14/79)*

Problem 121. *Does every hereditarily decomposable continuum contain an irreducible continuum with a composant whose complement is degenerate? (W. S. Mahavier, 03/14/79)*

Comment: Yes. (L. G. Oversteegen and E. D. Tymchatyn, 09/30/79)

Problem 122. *Suppose n is a positive integer and M is a continuum such that, for every positive number ϵ, there exists a weakly confluent ϵ-map of M onto an n-cell. Does M have the fixed point property? (H. Cook, 03/15/79)*

Comment: Yes for $n = 2$. (S. B. Nadler, *Fund. Math.* **110** (1980), pp. 231-232)

Problem 123. *Does there exist a widely connected complete metric space? (H. Cook, 03/16/79)*

Definition. A nondegenerate connected space X is **widely connected** if each nondegenerate connected subset of X is dense in X.

Problem 124. *Suppose M is the 'canonical' Knaster indecomposable continuum. Does M have a nonseparating closed set that intersects every composant of M? (H. Cook, 03/16/79)*

Comment: Yes. (W. Debski, 05/08/90)

Problem 125. *Does there exist a universal hereditarily indecomposable continuum? (H. Cook, 03/16/79)*

Comment: Yes. (T. Maćkowiak and P. Minc, 01/24/83)

Problem 126. *If M is an hereditarily indecomposable continuum containing a pseudo-arc P, is P a retract of M? (B. Knaster, 03/16/79)*

Comment: J. L. Cornette has shown that each subcontinuum the pseudo-arc is a retract of it. (H. Cook, 03/16/79)

Comment: If M is an hereditarily indecomposable continuum, K is a subcontinuum of M, and f is a mapping of K into a pseudo-arc P, then f can be extended to a mapping of M into P. (D. Bellamy, 05/05/79)

Problem 127. *Does a mapping of the plane into itself with bounded orbits have a fixed point? (H. Cook, 03/23/79)*

Comment: For mappings of the 3-dimensional Euclidean space, the answer is no. (K. Kuperberg and Coke Reed, *Fund. Math., Vol.* **114** (1981), p. 229)

Problem 128. *Given $n > 0$, is there a continuous of the $(2n+1)$-sphere into itself such that the orbit of each point is dense? (S. Fajtlowicz and D. Mauldin, 03/23/79)*

Problem 129. *Does there exist a chainable continuum M in the plane such that, if K is a chainable continuum in the plane, there exists a homeomorphism h of the plane onto itself that takes K into M? (R. H. Bing, 03/23/79)*

Problem 130. *Is it true that, if X is a continuum, f is a mapping of X into Hilbert space, and f has span zero, then f is almost factorable through $[0,1]$? (H. Cook, 03/23/79)*

Definition. Suppose X is a continuum, Y is a metric space, $\mathcal{C}$ is a collection of continua, and f is a mapping of X into Y. If ϵ is a positive number, f is **ϵ-almost factorable** through $\mathcal{C}$ if there exists a mapping k of X onto an element T of $\mathcal{C}$ and a mapping p of T into Y such that, for each point x of X, $d(f(x), pk(x)) \leq \epsilon$; and f is **almost factorable** through $\mathcal{C}$ if it is ϵ-almost factorable through $\mathcal{C}$ for every positive number ϵ. (M. Marsh, 03/23/79)

Problem 131. *Suppose M is a continuum such that, if G is an uncountable collection of nondegenerate subcontinua of M, then some two elements of G have a nondegenerate continuum in their intersection. Does M contain a countable point set that intersects every nondegenerate subcontinuum of M? (H. Cook, 03/26/79)*

Problem 132. *Is there an atriodic tree-like continuum which cannot be embedded in the plane? (W. T. Ingram, 03/27/79)*

Comment: Yes. (L. G. Oversteegen and E. D. Tymchatyn, 09/30/81)

Problem 133. *If M is an atriodic tree-like continuum in the plane, does there exist an uncountable collection of mutually exclusive continua in the plane each member of which is homeomorphic to M? (R. H. Bing, 03/27/79)*

Comment: No. (L. G. Oversteegen and E. D. Tymchatyn, 09/30/81)

Problem 134. *Is there an atriodic tree-like continuum M with positive span which has the property that there exists an uncountable collection G of mutually exclusive continua in the plane such that each member of G is homeomorphic to M? (W. T. Ingram, 03/27/79)*

Problem 135. *Suppose M is an hereditarily indecomposable simple triod-like continuum such that every proper subcontinuum of M is a pseudo-arc. Can M be embedded in the plane? (C. E. Burgess, 03/27/79)*

Problem 136. *Given a set X of n points on the plane, a line is* **ordinary** *if it contains exactly two points of X. A point is* **ordinary** *if it is on two ordinary lines. Is it true that, if not all points of X are on one line, then X contains an ordinary point? (S. Fajtlowicz, 03/28/79)*

Comment: No, for $n = 6$. (David Jones, 04/16/79)

Comment: What is the answer if n is odd? (S. Fajtlowicz, 04/16/79)

Problem 137. *Is there a monotonely refinable map from a regular curve of finite order onto a topologically different regular curve of finite order? (E. E. Grace, 04/11/79)*

Definition. A map r from a compact metric space X onto a compact metric space Y is **(monotonely) refinable** if, for each positive number ϵ, there is a (monotone) ϵ-map f from X onto Y such that, for each x in X, $d\left(f\left(x\right), r\left(x\right)\right) < \epsilon$.

Problem 138. *Does there exist, for every $k \leq m \leq \omega$, a space X such that:*

(1) *X^n is paracompact if and only if $n < m$, and*
(2) *X^n is Lindelöf if and only if $n < k$?*

(T. C. Przymusinski, 04/11/79)

Problem 139. *Does there exist a Lindelöf space X and a complete separable metric space M such that the product space $X \times M$ is not Lindelöf or (equivalently) normal? (T. C. Przymusinski, 4/11/79)*

Problem 140. *Is a para-Lindelöf space paracompact? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Problem 141. *Is a collectionwise normal space with a σ-locally countable base metrizable? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Problem 142. *Is a sL-cwH, metacompact space para-Lindelöf? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Definition. A space X is sL-cwH (strongly Lindelöf collectionwise Hausdorff) if every discrete collection of closed Lindelöf sets can be separated by a disjoint family of open sets.

Problem 143. *Does a para-Lindelöf space with a base of countable order have a σ-locally countable base? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Problem 144. *Is there an honest (i.e. provable from ZFC) example of a space of cardinality ω_1 with a σ-locally countable base which is not perfect? not metacompact? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Definition. A space is said to be **perfect** if every closed set is a G_δ.

Problem 145. *If X has a σ-disjoint base B and a σ-locally countable base B', must X have a base B'' which is simultaneously σ-disjoint and σ-locally countable? (W. G. Fleissner and G. M. Reed, 04/11/79)*

Problem 146. *(1) $a(i)(\mod n(i))$, $n(1) < n(2) < \cdots < n(k)$, is called* **covering** *if every integer satisfies at least one of the congruences (1). Let c be an arbitrary constant. Is there always a system (1) satisfying $n(1) > c$ which is a covering? (P. Erdős, 04/18/79)*

Comment: $1,000.00 for proof or disproof. Choi has, in *Math. of Computation*, a system with $n(1) = 20$. (P. Erdős, 04/18/79)

Problem 147. *Let there be given n points in the plane, no $n-k$ on a line. Join every two of the points. Prove that they determine at least $a \cdot k \cdot n$ distinct lines where a is an absolute constant independent of k and n. (P. Erdős, 04/18/79)*

Comment: $100.00 for proof or disproof. (P. Erdős, 04/18/79)

Comment: Proven. (J. Beck, *Combinatorica* **3** (1983), 281–293)

Problem 148. *Let $G(n)$ be a graph of n vertices. A Theorem of Goodman-Posa and myself states that the edges of our graph $G(n)$ can be covered by at most $n^2/4$ cliques, (i.e. maximal complete subgraphs). In fact the cliques can be assumed to be edge-disjoint and it suffices to use edges and triangles. Assume now that every edge of $G(n)$ is contained in a triangle. It seems that very much fewer than $n^2/4$ cliques will suffice to cover all edges. Determine the value of the least $g(n)$ so that the edges of such a graph can be covered by $g(n)$ cliques. (If too hard then try to determine $\lim g(n)/n^2$.) (P. Erdős, 04/18/79)*

Comment: The number of covering cliques is essentially the same; consider a complete tripartite graph in which two parts are equal and the third one consists of one element. (S. Fajtlowicz, 05/01/79)

Problem 149. *Is it true that the subset E of the complex plane is a type-A [type-B] convergence set if and only if $V_A(E)$ [$V_B(E)$] is bounded? (F. A. Roach, 04/18/79)*

Definition. A continued fraction is called a **type-A** [**type-B**] continued fraction provided every partial denominator [partial numerator] is unity. If E is a subset of the complex plane, $V_A(E)$ [$V_B(E)$] denotes the set of all values of finite type-A [type-B] continued fractions with elements from E. The set E is called a **type-A** [**type-B**] **convergence set** provided that every type-A [type-B] continued fraction with elements from E converges.

Comment: If 'complex plane' is replaced by 'real line', the resulting statements are true.

Problem 150. *Let $\mathcal{F}$ be a class of mappings such that all homeomorphism are in $\mathcal{F}$ and the composition of any two functions in $\mathcal{F}$ is also in $\mathcal{F}$. If X is homogeneous* *with respect to $\mathcal{F}$, is there a continuum which is $\mathcal{F}$-equivalent to X and which is homogeneous? (H. Cook, 04/25/79)*

Definition. A continuum X is **homogeneous** with respect to $\mathcal{F}$ if, for each two points a and b of X, there is a mapping f in $\mathcal{F}$ of X onto X such that $f(a) = b$.

Definition. If X and Y are continua, we say that Y is **$\mathcal{F}$-equivalent** to X provided there is a mapping in $\mathcal{F}$ from X onto Y and a mapping in $\mathcal{F}$ from Y onto X.

Comment: The answer is no for the class $\mathcal{F}$ consisting of all homeomorphisms and all mappings whose range is not homogeneous. (David Jones, 05/09/79)

Comment: What is the answer if $\mathcal{F}$ is the class of all mappings? All monotone mappings? All open mappings? All finite to one mappings? All confluent mappings? (H. Cook, 5/11/79)

Comment: No, if $\mathcal{F}$ is the class of all confluent mappings. (H. Kato, 08/09/84)

Comment:No, if $\mathcal{F}$ is the class of all mappings; take X to be a harmonic fan as in the paper of P. Krupski in *Houston J. Math.* **5** (1979), 345–356. (J. Prajs, 03/01/89)

Problem 151. *Suppose, in the previous problem, X is a homogeneous continuum, Y is a continuum which is $\mathcal{F}$-equivalent to X, and ϵ is a positive number. Does there exist a positive number δ such that if a and b are two points of Y such that the distance from a to b is less than δ, then there is a mapping f in $\mathcal{F}$ from Y onto Y such that $f(a) = b$ and no point of Y is moved a distance more than ϵ? (H. Cook, 04/25/79)*

Comment: If the answer to problem 150 is 'yes', then the answer to this problem is 'no'. (H. Cook, 04/25/79)

Problem 152. *Let M_n be the lattice consisting of 0, 1, and n mutually complementary elements and P_m—the lattice of all partitions of an m-element set. Let f be a function (1-1 function) of M_n into P_m. What is the probability that f is a homomorphism (an automorphism)? (S. Fajtlowicz, 05/20/79)*

Problem 153. *Suppose M is a continuum and f is a mapping of M onto itself such that, for each point x, M is irreducible from x to $f(x)$. Is there an essential mapping of M onto a circle? (H. Cook, 06/06/79)*

Problem 154. *Is the space of automorphisms of the pseudo-arc connected? (J. Krasinkiewicz, 11/14/79)*

Problem 155. *Are planar dendroids (arcwise connected continua) weakly chainable? (J. Krasinkiewicz, 11/14/79)*

Problem 156. *Let X be a nondegenerate homogeneous hereditarily decomposable continuum. Is it true that X is homeomorphic to a circle? (J. Krasinkiewicz, 11/14/79)*

Problem 157. *Does there exist a finite (countable) to one mapping from a hereditarily indecomposable continuum onto a hereditarily decomposable continuum? (J. Krasinkiewicz, 11/14/79)*

Problem 158. *Let X be a nondegenerate continuum such that there exists a continuous decomposition of the plane into elements homeomorphic to X. Must X be the pseudo-arc? (J. Krasinkiewicz, 11/14/79)*

Problem 159. *Does there exist a dendroid D such that the set E of end points is closed and each point of $D \setminus E$ is a ramification point? (J. Krasinkiewicz, 11/14/79)*

Comment: Yes. (J. Nikiel, 01/01/84)

Problem 160. *Let X be a continuum such that the cone over X embeds in Euclidean 3-space. Does X embed in the 2-sphere? (J. Krasinkiewicz, 11/14/79)*

Problem 161. *If the set function T is continuous for the Hausdorff continuum S, is true that S is T-additive? (D. Bellamy, 02/18/80)*

Comment: A bushel of *Extra Fancy Stayman Winesap* apples for a solution.
(D. Bellamy, 02/21/80)

Definition. Let S be a compact Hausdorff space. The set function T is defined by: For every subset A of S,

$$T(A) = S - \{\, p \in S : p \text{ has a continuum neighborhood } W \text{ which misses } S\,\}.$$

Definition. S is T-additive if and only if, for all pairs of closed sets A and B in S, $T(A + B) = T(A) + T(B)$.

Problem 162. *If T is continuous for the (Hausdorff) continuum S, is it true that the collection $\{\, T(p) \mid p \in S \,\}$ is a continuous decomposition of S such that the quotient space is locally connected? (D. Bellamy, 02/18/80)*

Definition. T is **continuous for** S means that T is a continuous function from the hyperspace of closed subsets of S to itself.

Problem 163. *If T is continuous for S and there is a point p in S such that $T(p)$ has nonempty interior, is S indecomposable? (D. Bellamy, 02/18/80)*

Problem 164. *If X and Y are indecomposable continua, is T idempotent on $X \times Y$? Even for only the closed sets in $X \times Y$? (D. Bellamy, 02/18/80)*

Definition. T is **idempotent** on a space S if $T\,(T\,(A)) = T(A)$ for all subsets A of S.

Problem 165. *If X and Y are indecomposable continua, $S = X \times Y$, and W is a subcontinuum of S with nonempty interior, is $T(W) = S$? (F. B. Jones, 02/18/80)*

Problem 166. *If X is a compact metric continuum, p is in X, $Y = X - \{p\}$, and $\beta(Y) - Y$ is an indecomposable continuum, must X be locally connected (connected im kleinen) at p? Is it true that $X = M \cup K$ where M is compact, p is not in M, and K is irreducible from some point q to p? (D. Bellamy, 02/18/80)*

Comment: $\beta(X)$ is the Stone-Čech compactification of X.

Problem 167. *Suppose M is a noncompact n-manifold for some $n > 1$ and M has no two-point compactification. Is $\beta(M) - M$ necessarily an aposyndetic continuum? (D. Bellamy, 02/18/80)*

Comment: This is true if M is Euclidean n-space.

Problem 168. *Does there exist a thin space with infinitely many points? (P. H. Doyle, 02/18/80)*

Definition. A topological space S is **thin** if and only if for each two homeomorphic subset A and B of S, there is a homeomorphism h of S onto itself such that $h(A) = B$.

Comment: The only known examples of thin spaces are finite spaces which are the product of a discrete space with an indiscrete space (i.e. a space S whose only open sets are S and the empty set).

Problem 169. *Is every aposyndetic, homogeneous, one-dimensional continuum locally connected? (J. T. Rogers, 07/22/80)*

Comment: No. (J. T. Rogers, 04/26/82)

Problem 170. *Do there exist even integers i and j such that, for every odd integer k and sequence $A_1, A_2, A_3, \cdots$ such that $A_1 = k$ and, for each n, $A_{n+1} - A_n$ is either i or j, some term of that sequence is prime? (J. T. Lloyd, 11/14/80)*

Comment: If the prime k-tuple conjecture of Hardy and Littlewood is true, then A_n is a prime infinitely often for any i and j. In case $i = 2$ and $j = 4$, infinitely many prime twins suffice. (P. Erdős, 11/24/81)

Problem 171. *Is every weakly chainable atriodic tree-like continuum chainable? (Lee Mohler, 04/16/81)*

Problem 172. *Does there exist a continuum M such that no monotone continuous image of M contains a chainable continuum? (H. Cook, 05/13/81)*

Problem 173. *Do there exist, in the plane, two simple closed curves J and C such that C in in the bounded complementary domain of J and the span of C is greater than the span of J? (H. Cook, 05/15/81)*

Problem 174. *In a topological space X, each set A has its derived set,*

$$A^d = \left\{ x \in X \;\middle|\; x \in \overline{A - \{x\}} \right\}.$$

Does there exist a T_0-space X and a subset A of X such that the second derived set $(A^d)^d$ is not closed in X? (A. Lelek, 05/15/81)

Problem 175. *In how many ways can one put k dominoes on an $n \times n$ chessboard so that each covers exactly two squares? (S. Fajtlowicz, 05/15/81)*

Comment: The answer is unknown for $k < 3$. (S. Fajtlowicz, 03/12/86)

Problem 176. *Suppose $X_1, X_2, X_3, \cdots$ is a sequence of positive numbers which converges to zero. Is there a set of positive measure which does not contain a set similar to $\{X_1, X_2, X_3, \cdots\}$? (P. Erdős, 11/24/81)*

Problem 177. *The equation $n! = m!\,k!$ $(m > k)$ is solved for $n = 10$, $m = 7$, $k = 6$, and also (trivially) if $n = k!$, $m = k! - 1$. Are there any other solutions? (D. Levine, 07/13/82)*

Problem 178. *Does there exist, for each t in $(\frac{1}{2}, 1)$, a simple triod $X(t)$ on the plane such that, with the natural metric, we have*

$$\frac{\sigma^*\big(X(t)\big)}{\sigma\big(X(t)\big)} = t?$$

(A. Lelek, 09/27/82)

Comment: $\sigma^*\left(X\left(t\right)\right)$ and $\sigma\left(X\left(t\right)\right)$ denote, respectively, the surjective span and the span of $X(t)$.

Problem 179. *Is there a tree-like continuum X on the plane such that X is weakly chainable but a subcontinuum of X is not? (P. Minc, 01/24/83)*

Problem 180. *Suppose X is an arcwise connected continuum with a free arc and such that, for each point p of X, there exists a homeomorphism $h\colon X \to X$ with $h(x) = x$ if and only if $x = p$. Is X a simple closed curve? (H. Cook, 10/31/83)*

Comment: No. (H. Gladdines, 06/15/94)

Problem 181. *Suppose X is an arcwise connected continuum with a free arc and with the fixed set property for monotone onto maps. Is X a simple closed curve? (Y. Ohsuda, 10/31/83)*

Definition. A topological space X is said to have the **fixed set property** for a certain class $\mathcal{C}$ of maps of X onto itself provided there exists, for each non-empty closed set A in X, a map f in $\mathcal{C}$ such that $f(x) = x$ if and only if x is in A.

Problem 182. *Is it true that if T and T' are trees with T' contained in T, then the surjective span of T is greater than or equal to one-half the width of T'? What about continua T and T' with T' contained in T? (A. Lelek, 10/12/84)*

Problem 183. *Let R be a space having a topological property (P). Is there an R-monolithic (locally R-monolithic) space with property (P), where (P) is one of the following properties: countable, second countable, Moore? (S. Iliadis, 02/08/85)*

Definition. Let R be a topological space. A space X is called **R-monolithic** if every mapping from X into R is constant.

Problem 184. *Does an open mapping always preserve span zero of continua? (E. Duda, 02/22/85)*

Comment: Yes. (K. Kawamura, 03/16/87)

Problem 185. *Is the product of two unicoherent continua always unicoherent? (E. Duda, 02/22/85)*

Comment: No. (A. García-Máynez and A. Illanes, 06/15/89)

Problem 186. *Characterize mappings $f\colon X \twoheadrightarrow Y$ such that if H is a proper subcontinuum of Y, then there is a proper subcontinuum K of X such that $f(K)$ contains H. (E. Duda, 02/22/85)*

Problem 187. *Does the class of approximable mappings coincide with that of the confluent ones? (W. Debski, 03/22/85)*

Definition. Suppose P is an inverse limit of an inverse limit sequence, where each factor space is $[0,1]$ and each bonding map is open. A mapping $f\colon P \to [0,1]$ is called **approximable** provided, for each i, there is an open map $f_i\colon I_i \to [0,1]$ such that the composite $f_i p_i$ (where p_i denotes the projection of P onto I_i) converges uniformly to f. A mapping $f\colon P \to Q$ between two such inverse limits P and Q is called **approximable** provided, for each j, the composite $q_j f$ is approximable (where q_j denotes the projection of Q onto I_j).

Problem 188. *Is each continuum of span zero continuously ray extendable? (W. T. Ingram, 10/14/85)*

Definition. A continuum M is **continuously ray extendable** provided, for each mapping $f\colon X \to M$ of a continuum X onto M and for each ray L such that $L \cup M$ is a compactification of L with remainder M, there exists a ray (closed half-line) R, with $R \cup X$ a compactification of R and X its remainder, such that f can be extended continuously to a mapping of $R \cup X$ onto $L \cup M$. (C. Wayne Proctor, 10/14/85)

Problem 189. *Does every strongly infinite-dimensional absolute G_δ separable metric space contain a G_δ subspace which is totally disconnected and hereditarily strongly infinite-dimensional? (A. Lelek, 11/04/85)*

Definition. A space is **hereditarily strongly infinite-dimensional** provided every non-empty subset is 0-dimensional or strongly infinite-dimensional.

Problem 190. *Is it true that if A is a subset of the Euclidean n-space, $\mathbb{R}^n$, and* $\dim A \le n-2$*, then every two points in $\mathbb{R}^n \setminus A$ can be joined by a 1-dimensional continuum contained in $\mathbb{R}^n \setminus A$? (J. Krasinkiewicz, 05/16/86)*

Problem 191. *Is it true that if A is a subset of $\mathbb{R}^n$, then there exists a subset B of $\mathbb{R}^n$ which contains A and is such that* $\dim B = \dim A$ *and* $\dim(\mathbb{R}^n \setminus B) = n - \dim B - 1$*? (J. Krasinkiewicz, 05/16/86)*

Comment: An affirmative solution of problem 191 would imply an affirmative solution of problem 190. (J. Krasinkiewicz, 05/16/86)

Problem 192. *Does there exist a hereditarily infinite-dimensional metric continuum with trivial shape? (J. Krasinkiewicz, 06/12/86)*

Problem 193. *Is it true that if X is a homogeneous finite-dimensional metric non-degenerate continuum with trivial shape, then X is 1-dimensional? (J. Krasinkiewicz, 06/12/86)*

Problem 194. *Let X be a homogeneous curve such that the rank, r, of the first Čech cohomology group of X with integer coefficients is finite. Is $r \leq 1$? (J. Krasinkiewicz, 06/12/86)*

Problem 195. *Let $X_1, X_2, \cdots$ be an inverse sequence of polyhedra with bonding maps $P_n \colon X_{n+1} \to X_n$ such that the inverse limit is a hereditarily indecomposable continuum. Let F_n be a continuous mapping of the 2-sphere into X_n such that F_n is homotopic to the composite $P_n[F_{n+1}]$ for $n = 1, 2, \ldots$. Is F_1 homotopic to a constant mapping? (J. Krasinkiewicz, 06/12/86)*

Problem 196. *Does there exist a piece-wise linear mapping f of a tree onto itself such that $\sigma(f^2) > 0$ and $\lim_{n\to\infty} \sigma(f^n) = 0$? (W. T. Ingram, 11/10/86)*

Notation. $\sigma(f^n)$ denotes the span of f^n

Comment: Yes. In fact, for each positive integer n there is a piece-wise linear mapping of a triod onto itself so that $\sigma(f^i) > 0$ for each $i \leq n$ and $\sigma(f^i) = 0$ for $i > n$. (S. W. Young, 11/02/89)

Problem 197. *Does there exist a separable metric space X whose dimension is greater than 1 and such that X satisfies condition (R_1)? (L. G. Oversteegen, 11/13/87)*

Definition. A topological space X satisfies **condition (R_1)** provided there exists a basis $\mathcal{B}$ of closed sets in X such that if U is in $\mathcal{B}$ and x is any point of $X \setminus U$, then X is not connected between $\{x\}$ and U.

Definition. A collection $\mathcal{B}$ of closed subsets of a topological space X is a **basis of closed sets** in X provided there exists, for each point p of X and each neighborhood V of p in X, a set U in $\mathcal{B}$ such that U is contained in V and p belongs to the interior of U in X.

Comment: No. (L. G. Oversteegen and E. D. Tymchatyn, 08/18/92)

Problem 198. *Is the property of being weakly chainable a Whitney property? (H. Kato, 11/14/88)*

Definition. A topological property P is called a **Whitney property** if whenever a continuum X a continuum X has property P, so does every point-inverse under a Whitney map from $C(X)$ to $\mathbb{R}$, where $C(X)$ is the hyperspace of subcontinua of X.

Problem 199. *If X is a tree-like continuum (or a dendroid), does $C(X)$ have the fixed point property? (H. Kato, 11/14/88)*

Comment: In the case of arc-like continua, this has been proven by J. Segal. (H. Kato, 11/14/88)

Problem 200. *Is it true that there do not exist expansive homeomorphisms on any Peano curve (that is, a locally connected 1-dimensional continuum, in particular, the Menger universal curve)? (H. Kato, 11/21/88)*

Definition. If X is a compact metric space, a homeomorphism f of X onto itself is called **expansive** provided there exists a number $c > 0$ such that, for any two different points x and y of X, there is an integer n (possibly negative) with $d(f^n(x), f^n(y)) > c$.

Comment: Yes. (K. Kawamura, H. M. Tuncali and E. D. Tymchatyn, 05/15/93)

Problem 201. *If C is a simple closed curve, is the span of C equal to the semi-span of C? (A. Lelek, 01/16/89)*

Comment: An affirmative answer to part (4) of problem 83 would establish an analogous result for simple triods. (A. Lelek, 01/16/89)

Problem 202. *Is each separable metric, hereditarily locally connected space of dimension less than or equal to 1? (E. D. Tymchatyn, 02/27/89)*

Problem 203. *Can the space of homeomorphisms of the Menger universal curve be embedded as the set of end-points of an $\mathbb{R}$-tree? (E. D. Tymchatyn, 02/27/89)*

Definition. An **$\mathbb{R}$-tree** is a separable metric, uniquely arcwise connected, locally arcwise connected space.

Comment: Yes. (K. Kawamura, L. G. Oversteegen and E. D. Tymchatyn, 10/15/93)

Problem 204. *For separable metric spaces, does condition (R_1) imply condition (R_2)? (L. G. Oversteegen and E. D. Tymchatyn, 04/14/89)*

Definition. A topological space X satisfies **condition (R_2)** provided there exists a basis $\mathcal{B}$ of closed sets in X such that if U and W are disjoint sets from $\mathcal{B}$, then X is not connected between U and W.

Comment: If the answer to problem 204 is yes, then the answer to problem 197 is no. (E. D. Tymchatyn, 04/14/89)

Comment: Yes. (L. G. Oversteegen and E. D. Tymchatyn, 08/18/92)

Problem 205. *Suppose X is a separable metric space and there exists a countable subset C of X such that $X \setminus C$ satisfies condition (R_2). Is $\dim X \leq 1$? (E. D. Tymchatyn, 04/14/89)*

Comment: An affirmative solution of problem 205 would imply an affirmative solution of problem 202. (E. D. Tymchatyn, 04/14/89)

Problem 206. *Are the sets of all end-points and of all branch-points (in the classical sense) of each dendroid Borel? (Analytic?) (A. Lelek and J. Nikiel, 04/21/89)*

Comment: The fact that the set of all end-points of each planar dendroid is $G_{\delta\sigma\delta}$ has been established in *Fund. Math.* **49** (1961), pp. 301–319. (A. Lelek, 04/21/89)

Comment: No, neither set need be Borel. However, the set of end-points is co-analytic and the set of branch-points is analytic. (J. Nikiel and E. D. Tymchatyn, 02/06/90)

Problem 207. *Can each n-dimensional, where $n > 1$, locally connected (metric) continuum be represented as the inverse limit of a sequence of n-dimensional polyhedra with monotone surjections as bonding maps? (J. Nikiel, 04/21/89)*

Comment: If $n = 1$, the answer is no. (J. Nikiel, 04/21/89)

Problem 208. *Suppose X is the inverse limit of a sequence of Hausdorff continua such that each of them is a continuous image of a Hausdorff arc and the bonding maps are monotone surjections. Is X also a continuous image of a Hausdorff arc? (J. Nikiel, 04/21/89)*

Comment: Yes. (J. Nikiel, H. M. Tuncali and E. D. Tymchatyn, 06/29/91)

Problem 209. *Let X be a Hausdorff space such that X is a continuous image of an orderable, compact Hausdorff space. Is X supercompact? (Regular supercompact?) (J. Nikiel, 04/21/89)*

Definition. A Hausdorff space X is **supercompact** if it admits a subbasis $\mathcal{S}$ (for open sets) such that each subcollection of $\mathcal{S}$ which is a covering of X contains a two-member subcollection which is a covering of X. The space X is **regular supercompact** provided, in addition, $\mathcal{S}$ has the property that if U is the union of a finite number of members of $\mathcal{S}$, then U is the interior of the closure of U in X.

Comment: It is known that if X is assumed, in addition, to be zero-dimensional or metrizable, then X is regular supercompact. A less general question was posed in 1977 by J. van Mill; he asked if each rim-finite continuum is supercompact. (J. Nikiel, 04/21/89)

Comment: Yes. (W. Bula, J. Nikiel, H. M. Tuncali and E. D. Tymchatyn, 11/15/90)

Problem 210. *Suppose X is a monotonically normal, compact space.*

(1) *Is X a continuous image of an orderable, compact Hausdorff space?*
(2) *Is X supercompact?*
(3) *Is X regular supercompact?*
(4) *What are the answers to the first three questions if, in addition, X is assumed to be zero-dimensional?*

(J. Nikiel, 04/21/89)

Comment: It is known that continuous images of orderable, compact Hausdorff spaces are monotonically normal. Moreover, P. Nyikos and S. Purisch proved in 1987 that monotonically normal, scattered, compact spaces are continuous images of well-ordered, compact Hausdorff spaces. Part (1) is related to the following problem raised in 1973 by S. Purisch: Is each monotonically normal, separable, zero-dimensional, compact space always orderable? (J. Nikiel, 04/21/89)

Problem 211. *Is each monotonically normal, compact space a continuous image of a monotonically normal, zero-dimensional, compact space? (J. Nikiel, 04/21/89)*

Problem 212. *Suppose X is a monotonically normal, separable, zero-dimensional, compact space. Is it true that if $\mathcal{C}$ is a collection of mutually disjoint closed subsets of X such that $\mathcal{C}$ is a null-family, then the collection of sets belonging to $\mathcal{C}$ which have at least* 3 *elements is countable? (J. Nikiel, 04/21/89)*

Definition. A collection $\mathcal{C}$ of subsets of a topological space X is a **null family** provided, for each open cover $\mathcal{U}$ of X, the collection of sets belonging to $\mathcal{C}$ which are not contained in any set from $\mathcal{U}$ is finite.

Problem 213. *Let X be a dendron. Does there exist a hereditarily indecomposable Hausdorff continuum Y such that X can be embedded in the hyperspace $C(Y)$ of subcontinua of Y? (J. Nikiel, 04/21/89)*

Definition. A Hausdorff continuum X is called a **dendron** provided, for every two distinct points x and y of X, there exists a point z of X such that x and y belong to distinct components of $X \setminus \{z\}$.

Comment: If X is a metrizable dendron, then X can be embedded in $C(Y)$ for each hereditarily indecomposable metric continuum Y. (J. Nikiel, 04/21/89)

Problem 214. *Does there exist a universal totally regular continuum? (J. Nikiel, 04/21/89)*

Definition. A continuum X is called **totally regular** provided X is metrizable and there exists, for each countable subset C of X, an open basis $\mathcal{B}$ in X such that the boundary of each set from $\mathcal{B}$ is finite and contained in $X \setminus C$.

Comment: Yes. (J. Buskirk, 10/11/91)

Problem 215. *Does there exist a continuous function f defined on the closed unit interval with values in a metric space such that f is at most n-to-one, for some positive integer n, and f is not finitely linear? (A. Lelek, 05/15/89)*

Definition. A function $f\colon I \to Y$, where I is a closed interval of the real line, is called **finitely linear** provided there exists a positive integer m such that I can be decomposed, for each $\epsilon > 0$, into a finite number of closed subintervals $I_1, I_2, \cdots, I_k$ each of length less than ϵ and with the property that the set $f(I_i)$ meets at most m of the sets $f(I_1), f(I_2), \cdots, f(I_k)$ for $i = 1, 2, \cdots, k$.

Definition. The usefulness of finitely linear functions has been shown in *Fund. Math.* **55** (1964), pp. 199–214. (A. Lelek, 05/15/89)

Problem 216. *Does there exist a monotone retraction $r\colon D \to C$ of the disk D onto its boundary C (that is, $r^{-1}(y)$ connected and $r(y) = y$ for each point y in C, and r not necessarily continuous) such that all but a finite number of points of C are values of continuity of r? (A. Lelek, 05/15/89)*

Definition. A point y in Y is called a **value of continuity** of a function $f\colon X \to Y$ provided $\lim_{n\to\infty} y_n = y$ implies that $\limsup f^{-1}(y_n)$ is contained in $f^{-1}(y)$ for any sequence $y_1, y_2, \cdots$ of points of Y.

Problem 217. *Does there exist, for every integer $n > 2$, a finite-dimensional separable metric space X and its finite-dimensional metric compactification cX such that if dX is a metric compactification of X and cX follows dX, then $\dim dX \geq n + \dim X$? (A. Lelek, 05/15/89)*

Definition. If cX and dX are compactifications of a topological space X, we say that cX **follows** dX provided there exists a continuous mapping $f\colon cX \to dX$ which commutes with the embeddings c and d, that is, $f \circ c = d$.

Comment: The answer is yes for $n = 2$. (A. Lelek, 05/15/89)

Problem 218. *Let G be the 3-dimensional unit cube in the Euclidean 3-space $\mathbb{R}^3$, and let A be a countable union of planes contained in $\mathbb{R}^3$ (not necessarily parallel). Suppose f is a continuous mapping of G into a metric space Y such that if $f^{-1}(y)$ is non-degenerate, for a point y in Y, then $f^{-1}(y)$ is contained in A. Is $\dim f(G) \geq 3$? (A. Lelek, 05/14/89)*

Comment: An affirmative answer to problem 218 implies an affirmative answer to problem 217 for $n = 3$. The relationship between these problems and some other similar compactification problems was described in the proceedings of a symposium: *Contributions to Extension Theory of Topological Structures*, Berlin 1969, pp. 147–148. The importance of compactifications is not undermined by the fact that they have been only sporadically discussed at the topology seminar of whose 18 years this book is record, more or less. (A. Lelek, 05/15/89)