1-1: Definitions and Termindogy Examples.

State the order of the given ODE. Determine whether the equation is linear or nonlinear by matching it with

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x).$$

1. $x\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0.$

3rd order.

Nonlinear since $\left(\frac{dy}{dx}\right)^4$

2. $\frac{d^2R}{dt^2} = -\frac{K}{R^2}$

2nd order.

Nonlinear since degree on $R \neq 1$.

3. $(\sin\theta)y''' - (\cos\theta)y' = 2$

3rd order

Linear

4. $(1+x)y'' - 4xy' + 5y = \cos x$

2nd order

Linear

Verify that the indicated function is an explicit solution of the given ODE.

5. $2y' + y = 0 \;;\; y(x) = e^{-x/2}$

RHS: 0

LHS: $2\left(\frac{-1}{2}\right)e^{-x/2} + e^{-x/2} = 0$

6. $\frac{dy}{dt} + 20y = 24 \;;\; y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

RHS: 24

LHS: $24e^{-20t} + 20\left(\frac{6}{5} - \frac{6}{5}e^{-20t}\right)$

$= 24e^{-20t} + 24 - 24e^{-20t}$

$= 24.$

7. $\frac{dP}{dt} = P(1-P)$; $P(t) = \frac{ce^t}{1+ce^t}$

LHS: $P'(t) = \frac{(1+ce^t)(ce^t)' - (ce^t)(1+ce^t)'}{(1+ce^t)^2}$

$= \frac{ce^t + c^2e^{2t} - c^2e^{2t}}{(1+ce^t)^2}$

$= \frac{ce^t}{(1+ce^t)^2}$

RHS: $P(1-P) = \left(\frac{ce^t}{1+ce^t}\right)\left(1 - \frac{ce^t}{1+ce^t}\right)$

$= \left(\frac{ce^t}{1+ce^t}\right)\left(\frac{1+ce^t - ce^t}{1+ce^t}\right)$

$= \frac{ce^t}{(1+ce^t)^2}$