1-2 IVP Examples.

Let $y(x) = \frac{1}{x^2+c}$ be a one-parameter family of solutions of the 1st order DE $y' + 2xy^2 = 0$. Find a solution of the the 1st order IVP consisting of this DE and a given IC.

1. $y(2) = 1/3$

$y(2) = \frac{1}{3} = \frac{1}{2^2+c}$

$c+4 = 3$

$c = -1$

So $y(x) = \frac{1}{x^2-1}$

2. $y(-2) = 1/2$

$y(-2) = \frac{1}{2} = \frac{1}{(-2)^2+c}$

$c+4 = 2$

$c = -2$

So $y(x) = \frac{1}{x^2-2}$

Let $x(t) = c_1 \cos t + c_2 \sin t$ be a two-parameter family of solutions of $x'' + x = 0$. Find a particular solution using the given IC.

3. $x(0) = -1$; $x'(0) = 8$

$x(0) = c_1 \cos(0) + c_2 \sin(0) = -1$

$c_1 = -1$

$x(t) = -\cos t + c_2 \sin t$

$x'(t) = \sin t + c_2 \cos t$

$x'(0) = \sin(0) + c_2 \cos(0) = 8$

$c_2 = 8$

$x(t) = -\cos t + 8 \sin t$

4. $x(\frac{\pi}{2}) = 0$; $x'(\frac{\pi}{2}) = 1$

$x(\frac{\pi}{2}) = c_1 \cos(\frac{\pi}{2}) + c_2 \sin(\frac{\pi}{2}) = 0$

$c_1(0) + c_2 = 0 \Rightarrow c_2 = 0$

$x(t) = c_1 \cos t$

$x'(t) = -c_1 \sin t$

$x'(\frac{\pi}{2}) = -c_1 \sin(\frac{\pi}{2}) = 1$

$-c_1 = 1 \Rightarrow c_1 = -1$

$x(t) = -\cos t$

Determine by inspection at least two solutions of the given 1st order ODE.

5. $xy' = 2y$; $y(0) = 0$

$$\frac{dy}{dx} = \frac{2y}{x}$$

Note $y=0$ is a solution since.

LHS: 0. (trivial solution)

RHS: 0.

Also $y = x^2$ is a solution since

LHS: $y' = 2x$

RHS: $\frac{2y}{x} = \frac{2x^2}{x} = 2x.$

6. $y' = 3y^{2/3}$; $y(0) = 0$

$y=0$ is a solution since

LHS: $y' = 0$

RHS: $3(0)^{2/3} = 0$

Also $y = x^3$ is a solution since

LHS: $y' = 3x^2$

RHS: $3y^{2/3} = 3(x^3)^{2/3} = 3x^2$

Determine a region of the xy-plane for the given DE would have a unique solution whose graph passes through the point (x_0, y_0) in the region.

7. $\frac{dy}{dx} = \sqrt{xy} = f(x,y)$

x: cont on $[0,\infty)$

y: cont on $[0,\infty)$.

$\frac{\partial f(x,y)}{\partial y} = \frac{\sqrt{x}}{2\sqrt{y}}$ x cont on $[0,\infty)$

y: cont on $(0,\infty)$.

Region where $x \geq 0$

$y > 0$.

8. $\frac{dy}{dx} = \frac{x^2}{1+y^3} = f(x,y)$

x cont everywhere.

y cont every where except $y=-1$

$$\frac{\partial f(x,y)}{\partial y} = \frac{(1+y^3)(x^2)' - x^2(1+y^3)'}{(1+y^3)^2}$$

$= \frac{-3x^2y^2}{(1+y^3)^2}$ x cont every where

y cont $(-\infty,-1) \cup (-1,\infty)$.

Region where x is anywhere

$-\infty < y < -1$ or $-1 < y < \infty$.