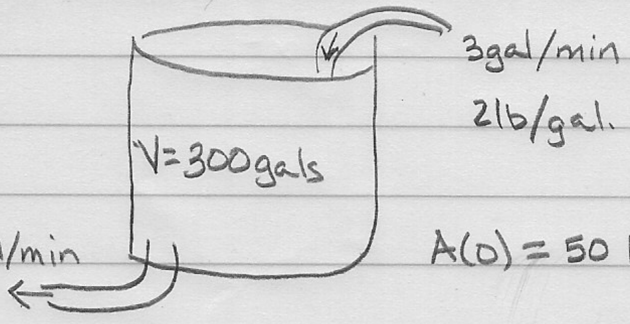
1-3 DEs as Mathematical Models Examples.

1. Suppose a student carrying a flu virus returns to an isolated college campus of 4000 students. Determine a DE for the number of people $x(t)$ who have contracted the flu if the rate at which the disease spread is proportional to the number of interactions between. the number of those infected and those that aren't.

Notice that we have a fixed population $n = 4000$.
Let y represent those that aren't infected.
Then $y + x = 4000$
So $\frac{dx}{dt} = Kxy$

$= Kx(4000 - x).$

2. Suppose that a tank holds 300 gals of H_2O initially where 50 lbs of salt have been dissolved. Another brine solution is pumped in at a rate of 3gal/min with a concentration of 2lb/gal. and pumped out at 2gal/min. Determine a DE for the amount of salt $A(t)$ in the tank at time t.



$A(0) = 50$ lb.

$$\frac{dA}{dt} = R_i - R_o$$

R_i = concentration · flow

$$= \left(\frac{2\,\text{lb}}{\text{gal}}\right)\left(\frac{3\,\text{gal}}{\text{min}}\right)$$

$$= \frac{6\,\text{lb}}{\text{min}}.$$

R_o = concentration · flow

$$= \left(\frac{A(t)}{300+t}\right)\left(\frac{2\,\text{gal}}{\text{min}}\right)$$

note: after t mins, the tank is filled t gals.

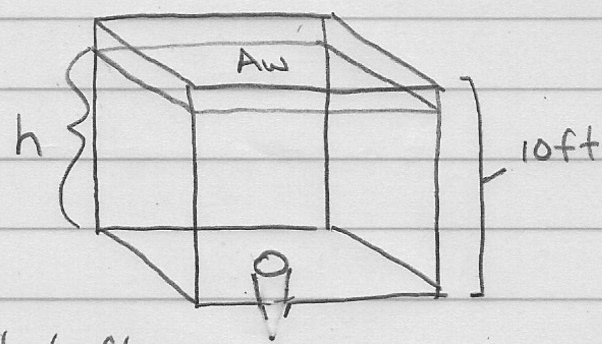
$$= \frac{2A(t)}{300+t}$$

Together with the IC, we have the IVP

$$\frac{dA}{dt} = 6 - \frac{2A}{300+t}$$

$$A(0) = 50.$$

3. Suppose H_2O is leaking from a tank through a circular hole of area A_h at its bottom. When H_2O leaks through a hole, friction and contraction of the stream near the hole reduce the volume of H_2O leaving the tank per second to $cA_h\sqrt{2gh}$, where c $(0<c<1)$ is an empirical constant. Determine a DE for the height h of H_2O at time t for a cubical tank where radius of the hole is 2 in. and $g = 32\,ft/s^2$.



$$\frac{dh}{dt} = \frac{-cA_h}{A_w}\sqrt{2gh}$$

$$A_h = \pi r^2 = \pi\left(\frac{2}{12}\right)^2 = \frac{\pi}{36}$$

need to convert to ft.

$$A_w = lw = 10^2 = 100\,ft^2$$

So
$$\frac{dh}{dt} = -c\left(\frac{\pi}{36}\right)\left(\frac{1}{100}\right)\sqrt{2(32)h}$$

$$= \frac{-c\pi}{3600}\sqrt{64h}$$

$$= \frac{-8c\pi\sqrt{h}}{3600}$$

$$= \frac{-c\pi}{450}\sqrt{h}.$$