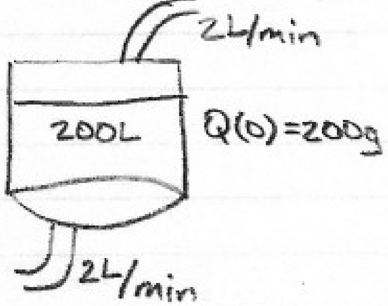
Sec 2-3 §1,6,7,9,14.

1. 

$$\frac{dQ}{dt} = R_i - R_o. \qquad R_i = \left(\frac{2L}{min}\right)(0) = 0$$

$$= -\frac{Q}{100} \qquad R_o = \left(\frac{2L}{min}\right)\left(\frac{Q}{200L}\right) = \frac{Q}{100}$$

$$\Rightarrow \int \frac{dQ}{Q} = -\int \frac{dt}{100}$$

$$\ln Q = \frac{-t}{100} + c$$

$$Q = ce^{-t/100}$$

$$200 = ce^0 \rightarrow c = 200$$

$$Q(t) = 200e^{-t/100}$$

$$2 = 200e^{-t/100}$$

$$t = -100\ln\left|\frac{1}{100}\right| = 100\ln(100) \approx 460.5 \text{min.}$$

6\. a. $S(t) = S_0e^{rt}$

$2S_0 = S_0e^{rT}$

$rT = \ln 2$

$T = \frac{\ln 2}{r}$

b. $T = \frac{\ln 2}{.07}$

≈ 9.90

c. $2S_0 = S_0e^{8r}$

$8r = \ln 2$

$r = \frac{\ln 2}{8} \approx 8.66\%.$

7\. $S(t) = \underbrace{S_0}_{=0}e^{rt} + \frac{k}{r}(e^{rt}-1)$

a. $S(t) = \frac{k}{r}(e^{rt}-1)$

b. $1{,}000{,}000 = \frac{K}{.075}\left(e^{.075(40)}-1\right)$

$K = \frac{75000}{(e^3-1)} \approx 3929.68$

c. $1{,}000{,}000 = \frac{200}{r}\left(e^{40r}-1\right)$

$5000r - e^{40r} + 1 = 0$

Maple $\rightarrow r \approx 0.0977$

9. $\begin{cases} S_0 = -8000 \\ r = .1 \\ t = 3 \end{cases}$

$$S(t) = S_0 e^{rt} + \frac{K}{r}(e^{rt} - 1)$$
$$0 = -8000e^{.1(3)} + \frac{K}{.1}(e^{.3} - 1)$$
$$10{,}798.9 = \frac{K}{.1}(e^{.3} - 1)$$
$$K = \frac{1079.89}{e^{.3} - 1} \approx \$3086.64/\text{yr.}$$

Interest: $3(3086.64) - 8000 = \$1259.92$.

14. a. $Q' = -rQ$

$$\int \frac{dQ}{Q} = -r\int dt$$
$$\ln Q = -rt + c$$
$$Q(t) = ce^{-rt}$$
$$\tfrac{1}{2}Q_0 = ce^{-5730r}$$
$$c = \tfrac{1}{2}Q_0 e^{5730r}$$
$$Q(t) = \tfrac{1}{2}Q_0 e^{5730r} e^{-rt}$$
$$.00236Q_0 = \tfrac{1}{2}Q_0 e^{5730r} e^{-50000r}$$
$$r = \frac{\ln(.00472)}{-44270} \approx .000121$$

b. $Q(t) = ce^{-rt}$

$$Q_0 = ce^0 \rightarrow c = Q_0$$
$$Q(t) = Q_0 e^{-rt}$$
$$\tfrac{1}{2}Q_0 = Q_0 e^{-5730r}$$
$$r = \frac{\ln \tfrac{1}{2}}{-5730} = .000121$$
$$Q(t) = Q_0 e^{-.000121t}$$

c. $.2Q_0 = Q_0 e^{-.000121t}$

$$t = \frac{\ln .2}{-.000121} \approx 13301.1 \text{ yr.}$$

Sec 2-4 8 1, 4, 11.

1. $(t-3)y' + (\ln t)y = 2t \quad y(1) = 2$ linear.

$y' + \left(\frac{\ln t}{t-3}\right) y = \frac{2t}{t-3} \rightarrow t \neq 3$

$\hookrightarrow t > 0, \, t \neq 3$

Then $t=1$ is in $(0,3)$.

so I is $0 < t < 3$.

4. $(4-t^2)y' + 2ty = 3t^2 \quad y(-3) = 1$

$y' + \left(\frac{2t}{4-t^2}\right) y = \frac{3t^2}{4-t^2}$

$\hookrightarrow t \neq -2, 2 \qquad \hookrightarrow t \neq -2, 2$

Then $t = -3$ is in $(-\infty, -2)$

11. $\frac{dy}{dt} = \frac{1+t^2}{3y-y^2} = \frac{1+t^2}{y(3-y)}$

$f(t,y)$ is cont when $\begin{cases} t \text{ is real} \\ y \neq 0, 3 \end{cases}$

$\frac{\partial f}{\partial y} = \frac{(1+t^2)(3-2y)}{(3y-y^2)^2}$ is cont when $\begin{cases} t \text{ is all reals} \\ y \neq 0, 3 \end{cases}$

Thus there is a unique solution for $\frac{dy}{dt}$ when

$y \neq 0$ or $y \neq 3$.