3.5 #1, 6, 11, 23, 25.

1. $y'' - 2y' + y = 0$

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$y(t) = c_1 e^t + c_2 te^t$

6. $y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$y(t) = c_1 e^{3t} + c_2 te^{3t}$

11. $9y'' - 12y' + 4y = 0 \qquad y(0) = 2 \quad y'(0) = -1.$

$9r^2 - 12r + 4 = 0$

$r = \frac{12 \pm \sqrt{144 - 4(4)(9)}}{18} = \frac{2}{3}$

$y(t) = c_1 e^{\frac{2}{3}t} + c_2 te^{\frac{2}{3}t}$

$y(0) = c_1 + 0 = 2 \quad \rightarrow c_1 = 2$

$y(t) = 2e^{\frac{2}{3}t} + c_2 te^{\frac{2}{3}t}$

$y'(t) = \frac{4}{3}e^{\frac{2}{3}t} + c_2 e^{\frac{2}{3}t} + \frac{2}{3}c_2 te^{\frac{2}{3}t}$

$y'(0) = \frac{4}{3} + c_2 = -1 \quad \rightarrow c_2 = -\frac{7}{3}$

$y(t) = 2e^{\frac{2}{3}t} - \frac{7}{3}te^{\frac{2}{3}t}$

Then $y(t) \to -\infty$ as $t \to \infty$.

23. $t^2y'' - 4ty' + 6y = 0 \;, \; t > 0 \;; \; y_1(t) = t^2$

$y_2 = ut^2$

$y_2' = u't^2 + 2ut$

$y_2'' = u''t^2 + 2u't + 2u't + 2u = u''t^2 + 4u't + 2u.$

$t^2y_2'' - 4ty_2' + 6y_2 = t^2(u''t^2 + 4u't + 2u) - 4t(u't^2 + 2ut) + 6ut^2$

$= u''t^4 + u'(4t^3 - 4t^3) + u(2t^2 - 8t^2 + 6t^2)$

$= u''t^4$

$= 0$

$u'' = 0$ let $\omega = u'$

$\omega' = 0$

$\omega = \int 0\,dt = c$

$u' = c$

$u = \int c\,dt = c_1 t + c_2 \Rightarrow u = t$

so $y_2 = ut^2 = t^3$

25. $t^2y'' + 3ty' + y = 0$, $t > 0$; $y_1(t) = t^{-1}$

$y_2 = ut^{-1}$

$y_2' = u't^{-1} - ut^{-2}$

$y_2'' = u''t^{-1} - u't^{-2} - u't^{-2} + 2ut^{-3} = u''t^{-1} - 2u't^{-2} + 2ut^{-3}$

$t^2y_2'' + 3ty_2' + y_2 = t^2(u''t^{-1} - 2u't^{-2} + 2ut^{-3}) + 3t(u't^{-1} - ut^{-2}) + ut^{-1}$

$= u''t + u'(-2 + 3) + u(2t^{-1} - 3t^{-1} + t^{-1})$

$= u''t + u'$

$= 0$

let $\omega = u'$

$t\omega' + \omega = 0$

$\omega' + \frac{1}{t}\omega = 0$ IF: $e^{\int \frac{1}{t}dt} = e^{\ln t} = t$

$t\omega' + \omega = 0$

$\frac{d}{dt}[t\omega] = 0$

$\int \frac{d}{dt}[t\omega]\,dt = 0$

$t\omega = c$

$\omega = \frac{c}{t} = u'$

$u = c\int \frac{1}{t}dt = c_1 \ln t + c_2 \Rightarrow u = \ln t$

$y_2 = ut^{-1} = t^{-1}\ln t.$