7-1 8 1, 11

1. Transform the given equation into a system of 1st order equations

$u'' + \frac{1}{2}u' + 2u = 0.$

$x_1 = u$

$x_2 = u' = x_1'$

$x_2' = u'' = -\frac{1}{2}u' - 2u = -\frac{1}{2}x_2 - 2x_1$

$$\begin{cases} x_1' = x_2 \\ x_2' = -\frac{1}{2}x_2 - 2x_1 \end{cases}$$

11. Transform the given system into a single equation of 2nd order. Then find x_1, x_2 that satisfy the given initial conditions.

$$\begin{cases} x_1' = 2x_2 & x_1(0) = 3 \\ x_2' = -2x_1 & x_2(0) = 4 \end{cases}$$

$x_2 = \frac{1}{2}x_1'$

$x_2' = \frac{1}{2}x_1''$

Then $\frac{1}{2}x_1'' = -2x_1 \Rightarrow \frac{1}{2}x_1'' + 2x_1 = 0$

$\Rightarrow x_1'' + 4x_1 = 0$

$\Rightarrow r^2 + 4 = 0$

$x_1 = c_1 \cos 2t + c_2 \sin 2t$

$x_2 = \frac{1}{2}(-2c_1 \sin 2t + 2c_2 \cos 2t)$

$= -c_1 \sin 2t + c_2 \cos 2t$

$x_1(0) = c_1 = 3$ $\quad$ $x_1(t) = 3\cos 2t + 4\sin 2t$

$x_2(0) = c_2 = 4$ $\quad$ $x_2(t) = 4\cos 2t - 3\sin 2t$

7-2 # 1, 4, 8, 10, 12, 21.

1. If $A=\begin{pmatrix}1 & -2 & 0\\ 3 & 2 & -1\\ -2 & 1 & 3\end{pmatrix}$ and $B=\begin{pmatrix}4 & -2 & 3\\ -1 & 5 & 0\\ 6 & 1 & 2\end{pmatrix}$, then

a. $2A+B=\begin{pmatrix}2 & -4 & 0\\ 6 & 4 & -2\\ -4 & 2 & 6\end{pmatrix}+\begin{pmatrix}4 & -2 & 3\\ -1 & 5 & 0\\ 6 & 1 & 2\end{pmatrix}=\begin{pmatrix}6 & -6 & 3\\ 5 & 9 & -2\\ 2 & 3 & 8\end{pmatrix}$

b. $A-4B=\begin{pmatrix}1 & -2 & 0\\ 3 & 2 & -1\\ -2 & 1 & 3\end{pmatrix}-\begin{pmatrix}16 & -8 & 12\\ -4 & 20 & 0\\ 24 & 4 & 8\end{pmatrix}=\begin{pmatrix}-15 & 6 & -12\\ 7 & -18 & -1\\ -26 & -3 & -5\end{pmatrix}$

c. $AB=\begin{pmatrix}1(4)-2(-1)+0(6) & 1(-2)-2(5)+0(1) & 1(3)-2(0)+0(2)\\ 3(4)+2(-1)-1(6) & 3(-2)+2(5)-1(1) & 3(3)+2(0)-1(2)\\ -2(4)+1(-1)+3(6) & -2(-2)+1(5)+3(1) & -2(3)+1(0)+3(2)\end{pmatrix}$

$=\begin{pmatrix}6 & -12 & 3\\ 4 & 3 & 7\\ 9 & 12 & 0\end{pmatrix}$

d. $BA=\begin{pmatrix}4(1)-2(3)+3(-2) & 4(-2)-2(2)+3(1) & 4(0)-2(-1)+3(3)\\ -1(1)+5(3)+0(-2) & -1(-2)+5(2)+0(1) & -1(0)+5(-1)+0(3)\\ 6(1)+1(3)+2(-2) & 6(-2)+1(2)+2(1) & 6(0)+1(-1)+2(3)\end{pmatrix}$

$=\begin{pmatrix}-8 & -9 & 11\\ 14 & 12 & -5\\ 5 & -8 & 5\end{pmatrix}$

7. If $A = \begin{pmatrix} 3-2i & 1+i \\ 2-i & -2+3i \end{pmatrix}$, then

a. $A^T = \begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$ b. $\bar{A} = \begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$

c. $A^* = \begin{pmatrix} 3+2i & 2+i \\ 1-i & -2-3i \end{pmatrix}$

8. If $\vec{x} = \begin{pmatrix} 2 \\ 3i \\ 1-i \end{pmatrix}$ and $\vec{y} = \begin{pmatrix} -1+i \\ 2 \\ 3-i \end{pmatrix}$, then

a. $x^T y = 2(-1+i) + 3i(2) + (1-i)(3-i) = -2+2i+6i+3-4i-1 = 4i$

b. $y^T y = (-1+i)^2 + 2^2 + (3-i)^2 = 1-2i-1+4+9-6i-1 = 12-8i$

c. $\langle x,y \rangle = 2(-1-i) + 3i(2) + (1-i)(3+i) = -2-2i+6i+3-2i+1 = 2+2i$

d. $\langle y,y \rangle = (-1+i)(-1-i) + 2(2) + (3-i)(3+i) = 1+1+4+9+1 = 16.$

10. $\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} = A$

$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array}\right]$, $R_2 \to R_2 + 2R_1$ $\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 11 & 2 & 1 \end{array}\right]$ $R_2 \to \frac{1}{11} R_2$ $\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right]$

$R_1 \to R_1 - 4R_2$ $\left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{11} & \frac{-4}{11} \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right]$

So $A^{-1} = \begin{bmatrix} \frac{3}{11} & \frac{-4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{bmatrix}$

12. $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array}\right]$$

$$-R_2 \leftrightarrow -R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right] \quad R_1 \to R_1 - 2R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -5 & 0 & 2 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 \to R_1 + 3R_3 \\ R_2 \to R_2 - 3R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

21. If $A(t) = \begin{pmatrix} e^t & 2e^{-t} & e^{2t} \\ 2e^t & e^{-t} & -e^{2t} \\ -e^t & 3e^{-t} & 2e^{2t} \end{pmatrix}$ and $B(t) = \begin{pmatrix} 2e^t & e^{-t} & 3e^{2t} \\ -e^t & 2e^{-t} & e^{2t} \\ 3e^t & -e^{-t} & -e^{2t} \end{pmatrix}$, then

a. $A + 3B = \begin{pmatrix} 7e^t & 5e^{-t} & 10e^{2t} \\ -e^t & 7e^{-t} & 2e^{2t} \\ 8e^t & 0 & -e^{2t} \end{pmatrix}$

b. $AB = \begin{pmatrix} e^t(2e^t) + 2e^{-t}(-e^t) + e^{2t}(3e^t) & e^t(e^{-t}) + 2e^{-t}(2e^{-t}) + e^{2t}(-e^{-t}) & e^t(3e^{2t}) + 2e^{-t}(e^{2t}) + e^{2t}(-e^{2t}) \\ 2e^t(2e^t) + e^{-t}(-e^t) - e^{2t}(3e^t) & 2e^t(e^{-t}) + e^{-t}(2e^{-t}) - e^{2t}(-e^{-t}) & 2e^t(3e^{2t}) + e^{-t}(e^{2t}) - e^{2t}(-e^{2t}) \\ -e^t(2e^t) + 3e^{-t}(-e^t) + 2e^{2t}(3e^t) & -e^t(e^{-t}) + 3e^{-t}(2e^{-t}) + 2e^{2t}(-e^{-t}) & -e^{-t}(3e^{2t}) + 3e^{-t}(e^{2t}) + 2e^{2t}(-e^{2t}) \end{pmatrix}$

$= \begin{pmatrix} 2e^{2t} - 2 + 3e^{3t} & 1 + 4e^{-2t} - e^t & 3e^{3t} + 2e^t - e^{4t} \\ 4e^{2t} - 1 - 3e^{3t} & 2 + 2e^{-2t} + e^t & 6e^{3t} + e^t + e^{4t} \\ -2e^{2t} - 3 + 6e^{3t} & -1 + 6e^{-2t} - 2e^t & -3e^{3t} + 3e^t - 2e^{4t} \end{pmatrix}$

c. $\frac{dA}{dt} = \begin{pmatrix} e^t & -2e^{-t} & 2e^{2t} \\ 2e^t & -e^{-t} & -2e^{2t} \\ -e^t & -3e^{-t} & 4e^{2t} \end{pmatrix}$

d. $\int_0^1 A(t)dt = \begin{pmatrix} e^t & -2e^{-t} & \frac{1}{2}e^{2t} \\ 2e^t & -e^{-t} & -\frac{1}{2}e^{2t} \\ -e^t & -3e^{-t} & e^{2t} \end{pmatrix}\Bigg|_0^1$

$$= \begin{pmatrix} (e-1) & (-2e^{-1}+2) & \frac{1}{2}(e^2-1) \\ (2e-2) & (-e^{-1}+1) & -\frac{1}{2}(e^{2t}-1) \\ (-e+1) & (-3e^{-1}+3) & (e^2-1) \end{pmatrix}$$

7-3 §2, 3, 7, 15, 21

2. $x_1 + 2x_2 - x_3 = 1$

$2x_1 + x_2 + x_3 = 1$

$x_1 - x_2 + 2x_3 = 1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & -1 & 2 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & -1 \\ 0 & -3 & 3 & 0 \end{array}\right]$$

$$R_3 \to R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & 1 \\ \underbrace{0 \quad 0 \quad 0}_{\text{zeros on left}} & & & 1 \end{array}\right]$$

zeros on left nonzero term on right → No Solution.

3. $x_1 + 2x_2 - x_3 = 2$

$2x_1 + x_2 + x_3 = 1$

$x_1 - x_2 + 2x_3 = -1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & -1 & 2 & -1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & -3 & 3 & -3 \end{array}\right]$$

$$\begin{array}{l} R_3 \to R_3 - R_2 \\ R_2 \to -\frac{1}{3}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 \to R_1 - 2R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $c_3 = t$

Then $c_2 - c_3 = 1 \Rightarrow c_2 = 1+t$

$c_1 + c_3 = 0 \Rightarrow c_1 = -t$

$$X = \begin{pmatrix} -t \\ 1+t \\ t \end{pmatrix}$$ where t is arbitrary

7. $X_1 = (2,1,0)$ $X_2 = (0,1,0)$ $X_3 = (-1,2,0)$

$$\det\begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = 0\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 0$$

linearly dependent.

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 \to R_2 - \tfrac{1}{2}R_1 \left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 0 & 1 & \frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $c_3 = t$ then $2c_1 = t \rightarrow c_1 = t/2$

$c_2 = \frac{-5}{2}t$

pick $c_3 = 2$ so $c_1 = 1$

$c_2 = -5$

Then $c_1X_1 + c_2X_2 + c_3X_3 = X_1 - 5X_2 + 2X_3 = 0$.

15 $\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$ $\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 5 + 3 = \lambda^2 - 6\lambda + 8$

$= (\lambda-2)(\lambda-4)$

For $\lambda_1 = 2$

$$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} 3p_1 - p_2 = 0 \\ 3p_1 - p_2 = 0 \end{matrix} \quad \begin{matrix} \text{let } p_2 = t \\ p_1 = \frac{1}{3}t \end{matrix} \quad \text{so } K_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

For $\lambda_2 = 4$

$$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} p_1 - p_2 = 0 \\ 3p_1 - 3p_2 = 0 \end{matrix} \quad \begin{matrix} \text{let } p_2 = t \\ p_1 = p_2 = t \end{matrix} \quad \text{so } K_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

21. $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$ $\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & 1-\lambda & -2 \\ 3 & 2 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix}$

$= (1-\lambda)[1-2\lambda+\lambda^2+4]$

$= (1-\lambda)(\lambda^2-2\lambda+5)$

$\lambda = \dfrac{2 \pm \sqrt{4-4(5)}}{2} = \dfrac{2 \pm 4i}{2} = 1 \pm 2i$

$\lambda_1 = 1$

$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ let $p_1 = t$

$2p_1 = 2p_3 \rightarrow p_3 = t$

$3p_1 = -2p_2 \rightarrow p_2 = -3/2\, t$

let $t = 2$

so $K_1 = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$

For $\lambda_2 = 1+2i$

$\begin{bmatrix} -2i & 0 & 0 \\ 2 & -2i & -2 \\ 3 & 2 & -2i \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $p_1 = 0$

$2ip_2 = 2p_3$

$2p_2 = 2ip_3$

scalar mults. let $p_3 = t$.

$K_2 = \begin{pmatrix} 0 \\ it \\ t \end{pmatrix}$ let $t = 1$ so $K_2 = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}$

Note $\lambda_3 = 1-2i = \bar{\lambda}_2$, so $K_3 = \bar{K}_2$

Then $K_3 = \begin{pmatrix} 0 \\ -i \\ 1 \end{pmatrix}$

7-4 #6,7

6. $x_1(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$, $x_2(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$

a. $W(x_1, x_2)(t) = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2$

b. x_1, x_2 are linearly independent everywhere except $t=0$

c. at least one of the coefficients is discontinuous at $t=0$.

d. $X = c_1 x_1 + c_2 x_2$

$X' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 2t \\ 2 \end{pmatrix} = \begin{pmatrix} c_1 + 2tc_2 \\ 2c_2 \end{pmatrix}$

$\Rightarrow c_2 = \frac{1}{2} x_2'$

$c_1 = x_1' - 2t\left(\frac{1}{2} x_2'\right) = x_1' - t x_2'$

Then $X = (x_1' - t x_2') \begin{pmatrix} t \\ 1 \end{pmatrix} + \frac{1}{2} x_2' \begin{pmatrix} t^2 \\ 2t \end{pmatrix} = \begin{pmatrix} x_1' t - \frac{1}{2} t^2 x_2' \\ x_1' \end{pmatrix} = \begin{pmatrix} t & -\frac{1}{2} t^2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$

Then $\left[\begin{array}{cc|cc} t & -\frac{1}{2} t^2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to R_1 - tR_2} \left[\begin{array}{cc|cc} 0 & -\frac{1}{2} t^2 & 1 & -t \\ 1 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_1 \to \frac{-2}{t^2} R_1} \left[\begin{array}{cc|cc} 0 & 1 & \frac{-2}{t^2} & \frac{2}{t} \\ 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{-2}{t^2} & \frac{2}{t} \end{array}\right]$

So $X' = \begin{bmatrix} 0 & 1 \\ \frac{-2}{t^2} & \frac{2}{t} \end{bmatrix} X$

7. $x_1(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$, $x_2(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$

a. $W(x_1, x_2)(t) = \begin{vmatrix} t^2 & e^t \\ 2t & e^t \end{vmatrix} = t^2e^t - 2te^t$

b. $t^2e^t - 2te^t = 0 \Rightarrow e^t(t^2 - 2t) = 0$

$\Rightarrow t(t-2) = 0$ linearly independent everywhere except $t=0$ and $t=2$.

c. at least one coefficient is discontinuous at $t=0$ and $t=2$.

d. $X = c_1 \begin{pmatrix} t^2 \\ 2t \end{pmatrix} + c_2 \begin{pmatrix} e^t \\ e^t \end{pmatrix}$

$X' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = c_1 \begin{pmatrix} 2t \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} e^t \\ e^t \end{pmatrix} = \begin{pmatrix} 2tc_1 + e^t c_2 \\ 2c_1 + e^t c_2 \end{pmatrix}$

$\left[\begin{array}{cc|c} 2t & e^t & x_1' \\ 2 & e^t & x_2' \end{array}\right]$ $R_1 \to R_1 - tR_2$ $\left[\begin{array}{cc|c} 0 & (1-t)e^t & x_1' - tx_2' \\ 2 & e^t & x_2' \end{array}\right]$ $R_1 \to \left[\frac{1}{(1-t)e^t}\right] R_1$

$\left[\begin{array}{cc|c} 0 & 1 & \frac{x_1' - tx_2'}{(1-t)e^t} \\ 2 & e^t & x_2' \end{array}\right]$ $R_2 \to R_2 - e^t R_1$ $\left[\begin{array}{cc|c} 0 & 1 & \frac{x_1' - tx_2'}{(1-t)e^t} \\ 2 & 0 & x_2' - \frac{x_1' - tx_2'}{(1-t)} \end{array}\right]$

Then $c_1 = \frac{x_2' - x_1'}{2(1-t)}$ $\quad c_2 = \frac{x_1' - tx_2'}{(1-t)e^t}$

$X = \left(\frac{x_2' - x_1'}{2-2t}\right)\begin{pmatrix} t^2 \\ 2t \end{pmatrix} + \left(\frac{x_1' - tx_2'}{(1-t)e^t}\right)\begin{pmatrix} e^t \\ e^t \end{pmatrix}$

$= \begin{bmatrix} \frac{t^2(x_2' - x_1')}{2-2t} + \frac{x_1' - tx_2'}{(1-t)} \\ \frac{t(x_2' - x_1')}{1-t} + \frac{x_1' - tx_2'}{1-t} \end{bmatrix} = \begin{bmatrix} \frac{2-t^2}{2-2t} & \frac{t^2-2t}{2-2t} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$

$$\left[\begin{array}{cc|cc} \frac{2-t^2}{2-2t} & \frac{t^2-2t}{2-2t} & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to R_1 - \left(\frac{2-t^2}{2-2t}\right) R_2} \left[\begin{array}{cc|cc} 0 & \frac{t^2-2t}{2-2t} & 1 & \frac{t^2-2}{2-2t} \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 \to \left(\frac{2-2t}{t^2-2t}\right) R_1 \left[\begin{array}{cc|cc} 0 & 1 & \frac{2-2t}{t^2-2t} & \frac{t^2-2}{t^2-2t} \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 \leftrightarrow R_2 \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{2-2t}{t^2-2t} & \frac{t^2-2}{t^2-2t} \end{array}\right]$$

so $X' = \begin{bmatrix} 0 & 1 \\ \frac{2-2t}{t^2-2t} & \frac{t^2-2}{t^2-2t} \end{bmatrix} X.$