

MTH204

Quiz 2

25 Jan 2008

Name: Key

Section C & F

Follow the directions carefully.
Please show all your work neatly
in pencil. This quiz is closed book,
closed notes, but you may use
your homework solutions. If
you get stuck, feel free to ask
me for help.

Consider the DE $(x-2)\ln y \frac{dy}{dx} = \frac{1}{3}y^{-2}$

a. Classify the DE by order and linearity.
Is the DE autonomous or separable? State why. What method(s) do you have to solve this DE?

1st order, nonlinear. Nonautonomous since x is in equation.
Separable since DE can be rewritten as $y' = g(y)h(x)$.
Method: Sol.

b. Solve the given DE.

$$3\int y^2 \ln y \, dy = \int \frac{dx}{(x-2)(x+3)}$$

$$\text{LHS: } u = \ln y \quad dv = y^2 \Rightarrow \int y^2 \ln y \, dy = \frac{1}{3}y^3 \ln y - \frac{1}{3} \int y^2 \, dy$$
$$du = \frac{1}{y} dy \quad v = \frac{1}{3}y^3$$

$$\text{RHS: } \frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \Rightarrow 1 = A(x+3) + B(x-2)$$

$$x=2 \Rightarrow 1 = 5A \Rightarrow A = \frac{1}{5}$$

$$x=-3 \Rightarrow 1 = -5B \Rightarrow B = -\frac{1}{5}$$

$$\Rightarrow 3 \left[\frac{1}{3}y^3 \ln y - \frac{1}{3} \int y^2 \, dy \right] = \frac{1}{5} \int \left(\frac{1}{x-2} - \frac{1}{x+3} \right) dx$$

$$y^3 \ln y - \frac{1}{3}y^3 = \frac{1}{5} (\ln|x-2| - \ln|x+3|) + C$$

$$y^3 (\ln y - \frac{1}{3}) = \frac{1}{5} \ln \left| \frac{x-2}{x+3} \right| + C$$