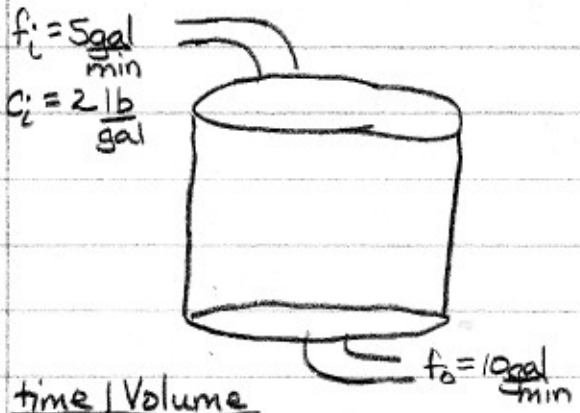
MTH 204
Quiz 3
1 Feb 2008

Name Key
Section C & F

Follow the directions carefully. Please show all work neatly in pencil and in the appropriate space. This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

A large tank is filled with 500 gal of pure water. Brine containing 2 lb salt per gal is pumped into the tank at a rate of 5 gal/min. The well mixed solution is pumped out at a rate of 10 gal/min.

a. Set up the IVP that describes the rate of change in the amount of salt, $A(t)$ at time t.



$$\frac{dA}{dt} = R_i - R_o$$

$$= 5(2) - 10\left(\frac{A}{500-5t}\right)$$

$$= 10 - \frac{2A}{100-t}$$

$A(0) = 0$

time	Volume
0	500
1	$500+5-10 = 500-5$
2	$500-5+5-10 = 500-5(2)$
⋮	
t	$500-5t$

b. Classify the DE. What method(s) do you have to solve this IVP?

1st order, linear, not autonomous, not separable, nonhomogeneous. (not in standard form).

Method: IF

c. Solve the IVP

$$\frac{dA}{dt} + \frac{2A}{100-t} = 10.$$

↳ looking for amount of salt, $A(t)$ where $t \in [0, 100)$.

$$IF = e^{\int P(t)dt} = e^{2\int \frac{dt}{100-t}} = e^{-2\ln|100-t|} = (100-t)^{-2}$$

$$(100-t)^{-2}\left[\frac{dA}{dt} + \frac{2A}{100-t}\right] = 10(100-t)^{-2}$$

$$(100-t)^{-2}\frac{dA}{dt} + \frac{2A}{(100-t)^3} = 10(100-t)^{-2}$$

$$\frac{d}{dt}\left[(100-t)^{-2}A\right] = 10(100-t)^{-2}$$

$$\int \frac{d}{dt}\left[(100-t)^{-2}A\right]dt = 10\int (100-t)^{-2}dt$$

$$(100-t)^{-2}A = 10(100-t)^{-1} + C$$

$$A(t) = 10(100-t) + C(100-t)^2$$

$$A(0) = 0 = 1000 + 100^2 C$$

$$\Rightarrow C = \frac{-1000}{100^2} = \frac{-1}{10}$$

$$A(t) = 10(100-t) - \frac{1}{10}(100-t)^2 \text{ for } t \in [0, 100).$$

Bonus (2pts): How can we change this tank problem so that we get a separable DE? Be specific. Make flow in equal to the flow out.