MTH204
Quiz 4
8 Feb 2008

Name: Key
Section C & F

Follow the directions carefully. Please show all your work neatly in pencil. This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help

Consider the DE $x^2y'' - xy' + y = 0$, $x > 0$

a. Let $f(x) = x$ and $g(x) = x^2$. Do $f(x)$ and $g(x)$ form a fundamental set of solutions for the given DE on $x > 0$? State why or why not.

$f(x) = x \quad f'(x) = 1 \quad f''(x) = 0$

$\Rightarrow x^2(0) - x(1) + x = 0 \quad \Rightarrow f(x)$ is a solution

$g(x) = x^2 \quad g'(x) = 2x \quad g''(x) = 2$

$\Rightarrow x^2(2) - x(2x) + x^2 = x^2 \neq 0 \Rightarrow g(x)$ is not a solution

$\therefore f(x)$ and $g(x)$ do not form a FSS for the given DE since $g(x)$ is not a solution.

b. Let $f(x) = x$ and $h(x) = x\ln x$. Do $f(x)$ and $h(x)$ form a fundamental set of solutions for the given DE on $x > 0$? State why or why not.

$h(x) = x\ln x \quad h'(x) = \ln x + 1 \quad h''(x) = x^{-1}$

$\Rightarrow x^2(x^{-1}) - x(\ln x + 1) + x\ln x = 0 \Rightarrow h(x)$ is a solution

$$W(f,h) = \begin{vmatrix} x & x\ln x \\ 1 & \ln x + 1 \end{vmatrix} = (x\ln x + x) - x\ln x = x \neq 0 \Rightarrow f, h \text{ are LI.}$$

$\therefore f(x)$ and $h(x)$ form a FSS for the given DE since they are linearly independent solutions where # LI solutions = order of DE.

Bonus (2 pts): What's the difference between a general solution and a superposition? Be specific.