MTH 204
Quiz 5
3 Oct 2008

Name Key
Section A&C

Follow the directions carefully. This quiz is closed book, closed notes, but you may use your homework solutions. Please write in pencil. You must show all your work to receive full credit. If you get stuck, feel free to ask me for help.

Thurs: LEAD
5-7, CSFG5D

Consider the DE $y''-5y'+4y=6e^x+12\cos(x)$

a. Classify the DE

2nd order, linear, nonhomogeneous, constant coefficients

b. What method(s) do we have to solve this DE?

Methods { MUC ✓
{ VOP

c. Solve the DE.

1. Solve $y''-5y'+4y=0$

$y(x)=e^{rx} \Rightarrow e^{rx}[r^2-5r+4]=0$

$\Rightarrow (r-1)(r-4)=0$

$\Rightarrow r=4,1$

$\Rightarrow y_h(x)=c_1e^{4x}+c_2e^x$

2. Find roots of $g(x)$

$r=1,\pm i$

3. Find a characteristic equation for $g(x)$.

$(r-1)(r^2+1)=0$

4. Find the annihilator of $g(x)$

$(D-1)(D^2+1)$

5. Apply the annihilator to both sides

$(D-1)(D^2+1)(D^2-5D+4)y=(D-1)(D^2+1)g(x)=0$

5th, linear, Hom, CC

6. Solve $\Rightarrow y(x)=e^{rx}$

$\Rightarrow e^{rx}(r-1)(r^2+1)(r-1)(r-4)=0$

$\Rightarrow r=4,1,1,\pm i$

$$\Rightarrow y(x) = \underbrace{c_1 e^{4x} + c_2 e^{x}}_{y_h} + \underbrace{c_3\cos(x) + c_4\sin(x) + c_5 x e^{x}}_{y_p}$$

7. Find $y_p$

$y_p(x) = A\cos(x) + B\sin(x) + Cxe^x$

$y_p'(x) = -A\sin(x) + B\cos(x) + Ce^x + Cxe^x$

$y_p''(x) = -A\cos(x) - B\sin(x) + 2Ce^x + Cxe^x$

8. MUC

$y_p'' - 5y_p' + 4y_p = (-A - 5B + 4A)\cos(x) + (-B + 5A + 4B)\sin(x)$
$+ (C - 5C + 4C)xe^x + (2C - 5C)e^x$
$= (3A - 5B)\cos(x) + (5A + 3B)\sin(x) - 3Ce^x$
$= 6e^x + 12\cos(x)$

$\Rightarrow \begin{cases} 3A - 5B = 12 \\ 5A + 3B = 0 \Rightarrow A = -\frac{3}{5}B \end{cases} \Rightarrow -\frac{9}{5}B - \frac{25}{5}B = 12$

$\Rightarrow B = -\frac{30}{17}, \quad A = \frac{18}{17}$

$-3C = 6 \Rightarrow C = -2$

$\Rightarrow y_p(x) = \frac{18}{17}\cos(x) - \frac{30}{17}\sin(x) - 2xe^x$

9. GS:

$y(x) = c_1 e^{4x} + c_2 e^{x} + \frac{18}{17}\cos(x) - \frac{30}{17}\sin(x) - 2xe^x$

Bonus (2pts): What is the annihilator of $g(x) = 6e^{-x}\cos(2x) + x\sin(2x) - 3x + 1 + 13e^{2x}$?

Roots: $r = -1 \pm 2i, \pm 2i, \pm 2i, 0, 0, 2$

Char Eq: $[(r+1)^2 + 4](r^2+4)^2 r^2 (r-2) = 0$

Ann: $[(D+1)^2 + 4](D^2+4)^2 D^2 (D-2)$