MTH 204
Quiz 5
22 Feb 2008

Name Key
Section C&F

Please follow the directions carefully.
Show all your work neatly in pencil.
This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

Solve $y''+6y'+8y = 3e^{-2x}+2x$ using MUC.

1. Solve $y''+6y'+8y=0$

Assume $y(x)=e^{rx} \Rightarrow e^{rx}[r^2+6r+8]=0$

$\Rightarrow (r+2)(r+4)=0$

$\Rightarrow y_h = c_1e^{-4x}+c_2e^{-2x}$

2. Identify roots of $g(x)$.

$r=-2,0,0$

3. Create a characteristic eq that gives $g(x)$

$r^2(r+2)=0$

4. Find an annihilator of $g(x)$

Ann: $D^2(D+2)$

5. Apply Ann to both sides

$D^2(D+2)(D+2)(D+4)\,y = D^2(D+2)[g(x)]=0$

5th order, linear, hom, CC

6. Solve. Assume $y(x)=e^{rx}$

$\Rightarrow e^{rx}r^2(r+2)(r+2)(r+4)=0$

$r=-4,-2,-2,0,0$

$\Rightarrow$ GS: $y(x) = \underbrace{c_1e^{-4x}+c_2e^{-2x}}_{y_h} + \underbrace{c_3xe^{-2x}+c_4+c_5}_{y_p}$

7. Identify y_p

$y_p = Axe^{-2x}+B+Cx$

$y_p' = Ae^{-2x}-2Axe^{-2x}+C$

$y_p'' = -4Ae^{-2x}+4Axe^{-2x}$

8. MUC

$$y_p'' + 6y_p' + 8y_p = (4A - 12A + 8A)xe^{-2x} + (-4A + 6A)e^{-2x} + (8C)x + (6C + 8B)$$
$$= 2Ae^{-2x} + 8Cx + (6C + 8B)$$
$$= 3e^{-2x} + 2x$$

Comparing coefficients

$$2A = 3 \Rightarrow A = \frac{3}{2}$$

$$8C = 2 \Rightarrow C = \frac{1}{4}$$

$$6C + 8B = 0 \Rightarrow B = -\frac{6}{8}C = -\frac{3}{16}$$

$$\Rightarrow y_p = \frac{3}{2}xe^{-2x} - \frac{3}{16} + \frac{x}{4}$$

9. GS: $y(x) = y_h(x) + y_p(x)$

$$= C_1e^{-4x} + C_2e^{-2x} + \frac{3}{2}xe^{-2x} - \frac{3}{16} + \frac{x}{4}$$