MTH 204
Quiz 6
10 Oct 2008

Name Key
Section A & C

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
to receive full credit. This quiz
is closed book, closed notes, but
you may use your homework
solutions. If you get stuck, feel
free to ask me for help.

LEAD - Thursdays
5-7 CSF G5D

mst.edu/~njwn7d.

Consider the DE $x^2y'' - 4xy' = x^5$, $x>0$

a. Classify the DE.

2nd order, linear, VC, NH (Cauchy-Euler)

b. What method(s) can you use to solve this DE?

Methods { ~~MUC~~ → "not in this form"
VOP

c. Note that $y_h(x) = c_1 + c_2x^5$. Find the general solution.

1. Solve $x^2y'' - 4xy' = 0$

Assume $y(x) = x^m$

$\Rightarrow y_h(x) = c_1y_1 + c_2y_2 = c_1 + c_2x^5$

2. Put DE in Std form

$$y'' - \frac{4}{x}y' = \underbrace{x^3}_{f(x)}$$

3. Find $W(y_1, y_2)$

$$W(y_1, y_2) = \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix} = 5x^4 - 0 = 5x^4$$

4. Assume $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$

"5" Plug y_p into DE

6. Cramer's Rule

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & x^5 \\ x^3 & - \end{vmatrix} = 0 - x^8 = -x^8$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ - & x^3 \end{vmatrix} = x^3 - 0 = x^3$$

So $u_1'(x) = \frac{W_1}{W(y_1, y_2)} = \frac{-x^8}{5x^4} = \frac{-1}{5}x^4$

and $u_2'(x) = \frac{W_2}{W(y_1,y_2)} = \frac{x^3}{5x^4} = \frac{1}{5x}$

7. Integrate & plug in.

$u_1(x) = -\frac{1}{5}\int x^4 dx = -\frac{1}{25}x^5 + \cancel{K_1}$

$u_2(x) = \frac{1}{5}\int \frac{dx}{x} = \frac{1}{5}\ln x + \cancel{K_2}$

$\Rightarrow y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x) = -\frac{1}{25}x^5(1) + \left(\frac{1}{5}\ln x\right)x^5$

↳ absorbed into y_2

8. Check if any terms are absorbed.

$y_p(x) = \frac{x^5}{5}\ln x$

9. GS: $y(x) = y_h(x) + y_p(x) = c_1 + c_2x^5 + \frac{x^5}{5}\ln x$

Bonus (3pts): A 16 lb weight stretches a spring $\frac{8}{3}$ ft. Initially the weight starts from rest 2 ft below the equilibrium position, and the subsequent motion in a medium offers a damping force numerically equal to $\frac{1}{2}$ the instantaneous velocity. Set up, but <u>don't</u> solve the IVP when there is an external force of $g(t) = 10\cos(3t)$.

$m = \frac{W}{g} = \frac{16}{32} = \frac{1}{2}$ slugs

$\beta = \frac{1}{2}$

$mg = Ks \Rightarrow 16 = \frac{8}{3}K$

$\Rightarrow K = 6$ lb/ft

IVP:

$\frac{1}{2}y'' + \frac{1}{2}y' + 6y = 10\cos(3t)$

$\begin{cases} y(0) = 2 \\ y'(0) = 0. \end{cases}$