MTH 204
Quiz 6
29 Feb 2008

Name Key
Section C & F

Follow the directions carefully. Please all your work neatly in pencil. This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

Consider the DE $4y'' - y = xe^{\frac{1}{2}x}$

a. Classify the DE

2nd order, linear, nonhomogeneous, constant coefficients (not in standard form)

b. What method(s) can you use to solve DE?

MUC & VOP

c. Use VOP to solve the DE.

1. Solve $4y'' - y = 0 \Rightarrow y(x) = e^{rx}$

$\Rightarrow e^{rx}[4r^2 - 1] = 0$

$\Rightarrow r = \pm 1/2$

$\Rightarrow y_h(x) = c_1 e^{1/2x} + c_2 e^{-1/2x}$

2. Find $W(y_1, y_2)$

$$W(y_1, y_2) = \begin{vmatrix} e^{1/2x} & e^{-1/2x} \\ \frac{1}{2}e^{1/2x} & -\frac{1}{2}e^{-\frac{1}{2}x} \end{vmatrix} = -\frac{1}{2} - \frac{1}{2} = -1$$

3. Put DE in Std form: $y'' - \frac{1}{4}y = \frac{1}{4}xe^{1/2x}$

4. Assume $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$.

"5" Plug in y_p

6. Cramer's Rule.

$$W_1 = \begin{vmatrix} 0 & e^{-1/2x} \\ \frac{1}{4}xe^{1/2x} & - \end{vmatrix} = 0 - \frac{1}{4}x = \frac{-x}{4}$$

$$W_2 = \begin{vmatrix} e^{1/2x} & 0 \\ - & \frac{1}{4}xe^{1/2x} \end{vmatrix} = \frac{1}{4}xe^{x} - 0 = \frac{xe^x}{4}$$

$$\Rightarrow u_1'(x) = \frac{W_1}{W(y_1, y_2)} = \frac{x}{4}$$

$$u_2'(x) = \frac{W_2}{W(y_1, y_2)} = -\frac{xe^x}{4}$$

7. Integrate & plug into (4).

$$u_1(x) = \frac{1}{4}\int x\,dx = \frac{1}{8}x^2 + K_1 \quad \hookrightarrow 0$$

$s=x\ dt=e^x dx$ $ds=dx\ t=e^x$

$$u_2(x) = -\frac{1}{4}\int x e^x dx = -\frac{1}{4}\left[xe^x - \int e^x dx\right] = -\frac{1}{4}\left[xe^x - e^x\right] + K_2 \quad \hookrightarrow 0$$

$$\Rightarrow y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x) = \frac{1}{8}x^2 e^{\frac{1}{2}x} - \frac{1}{4}(xe^x - e^x)e^{-\frac{1}{2}x}$$

$$= \frac{1}{8}x^2 e^{1/2x} - \frac{1}{4}xe^{1/2x} + \frac{1}{4}e^{1/2x}$$

8. Check if any terms of y_p are absorbed into y_h

$$y_p = \frac{1}{8}x^2 e^{1/2x} - \frac{1}{4}xe^{1/2x} + \frac{1}{4}e^{1/2x}$$

$\frac{1}{4}e^{1/2x}$ → absorbed by y_1

$$= \frac{1}{8}x^2 e^{1/2x} - \frac{1}{4}xe^{1/2x}$$

9. General Solution: $y(x) = y_h(x) + y_p(x)$

$$= c_1 e^{1/2x} + c_2 e^{-1/2x} + \frac{1}{8}x^2 e^{\frac{1}{2}x} - \frac{1}{4}xe^{1/2x}$$

<u>Bonus (3pts)</u>: Setup, but don't solve, an IVP for an LRC series circuit with an inductance of 0.25h, a resistance of 10Ω, a capacitance of 0.001f, and an impressed voltage of 200V. Assume the initial charge is 5C.