

MTH 204

Quiz 7

17 Oct 2008

Name _____ Key _____

Section A&C

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
to receive full credit. This quiz
is closed book, closed notes, but
you may use your homework
solutions. If you get stuck, feel
free to ask me for help.

LEAD - Thursdays
5-7 CSF G5D

mst.edu/~njwn7d.

1. Solve the following Cauchy-Euler equations for $x > 0$

a. $x^2 y'' - 5xy' + 9y = 0$.

Assume $y(x) = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$\Rightarrow x^m [m(m-1) - 5m + 9] = 0$$

$$m^2 + (-5-1)m + 9 = 0$$

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$\Rightarrow m = 3, 3$$

$$\Rightarrow y(x) = C_1 x^3 + C_2 x^3 / \ln x$$

b. $4x^2 y'' + 8xy' + 17y = 0$

Assume $y(x) = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$\Rightarrow x^m [4m(m-1) + 8m + 17] = 0$$

$$4m^2 + (8-4)m + 17 = 0$$

$$4m^2 + 4m + 17 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 4(4)(17)}}{2(4)} = \frac{-4 \pm \sqrt{-16^2}}{8} = \frac{-4 \pm 16i}{8} = -1 \pm 2i$$

$$\Rightarrow y(x) = x^{-1/2} [C_1 \cos(2 \ln x) + C_2 \sin(2 \ln x)]$$

2 Use the definition $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$ to find the Laplace transform of $f(t) = te^{3t}$. For what values of s does $\mathcal{L}\{f(t)\} = F(s)$ exist?

$$\mathcal{L}\{te^{3t}\} = \int_0^{\infty} e^{-st} te^{3t} dt$$

$$= \int_0^{\infty} e^{-(s-3)t} t dt$$

$$\begin{array}{ll} u = t & dv = e^{-(s-3)t} \\ du = dt & v = -\frac{1}{s-3} e^{-(s-3)t} \end{array}$$

$$= -\frac{t}{s-3} e^{-(s-3)t} \Big|_0^{\infty} + \frac{1}{s-3} \int_0^{\infty} e^{-(s-3)t} dt$$

$$= \underbrace{0-0}_{s-3>0} - \frac{1}{(s-3)^2} e^{-(s-3)t} \Big|_0^{\infty}$$

$$= 0 - - \frac{1}{(s-3)^2}$$

$$= \frac{1}{(s-3)^2} \quad \text{provided } s-3 > 0 \\ s > 3$$