

MTH204

Quiz 7

7 March 2008

Name Key

Section C&F

Follow the directions carefully.
Please show all your work neatly
in pencil. This quiz is closed notes,
closed book, but you may use your
homework solutions. If you get
stuck, feel free to ask me for
help.

1. Suppose an object weighing 24 lb stretches a vertical spring 3 ft when it is first attached. Initially the object is released 5 ft below the equilibrium position with an upward velocity of 1 ft/s. Assume there is no damping force, but there is an external forcing function of $f(t) = 10 \cos(\gamma t)$. Use $g = 32 \text{ ft/s}^2$.

a. Set up, but do not solve the IVP that models the object's displacement.

$$m = \frac{W}{g} = \frac{24}{32} = \frac{3}{4} \quad \beta = 0$$

Eg Position: $mg = Ks \Rightarrow 24 = 3K \Rightarrow K = 8$

$$\Rightarrow \frac{3}{4}y'' + 8y = 10 \cos(\gamma t)$$

$$\begin{cases} y(0) = 5 \\ y'(0) = -1 \end{cases}$$

b. For what value of γ will pure resonance occur?

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{8}{3/4}} = 4\sqrt{\frac{2}{3}} = \gamma.$$

2. Solve $x^2 y'' + 7xy' + 9y = 0, x > 0$

Assume $y(x) = x^m, y'(x) = mx^{m-1}, y''(x) = m(m-1)x^{m-2}$

$$\Rightarrow x^2 m(m-1)x^{m-2} + 7x mx^{m-1} + 9x^m = 0$$

$$x^m [m(m-1) + 7m + 9] = 0$$

$$m^2 + 6m + 9 = (m+3)^2 = 0 \Rightarrow m = -3, -3$$

$$\Rightarrow y(x) = c_1 x^{-3} + c_2 x^{-3} \ln x.$$