MTH 204 Name Key
Quiz 8 Section A+C
24 Oct 2008

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
to receive full credit. This quiz
is closed book, closed notes, but
you may use your homework
solutions. If you get stuck, feel
free to ask me for help.
A copy of the Laplace transform
table is on the back page.

LEAD - Thursdays
5-7 CSF G5D

Exam 2 - Oct 31
Sec: 4.3 - 7.3, 7.5

mst.edu/~gjwn7d.

Consider the IVP $y''-y'=5e^t\cos(2t)$, $\begin{cases} y(0)=3 \\ y'(0)=0 \end{cases}$

a. Classify the DE.

2nd order, linear, constant coefficients, nonhomogeneous.

b. Use the Laplace transform to solve the IVP.

1. Take the Laplace of both sides

$$\mathcal{L}\{y''-y'\} = \mathcal{L}\{5e^t\cos(2t)\}$$

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} = 5\,\mathcal{L}\{\cos(2t)\}_{s\to s-1}$$

$$\underbrace{(s^2Y(s)-sy(0)-y'(0))}_{3,\ 0} - \underbrace{(sY(s)-y(0))}_{3} = \frac{5s}{s^2+4}\Big|_{s\to s-1}$$

$$(s^2-s)Y(s) - 3(s-1) = \frac{5(s-1)}{(s-1)^2+4}$$

2. Solve for $Y(s)$

$$s(s-1)Y(s) = \frac{5(s-1)}{(s-1)^2+4} + 3(s-1)$$

$$\Rightarrow Y(s) = \frac{5(s-1)}{s(s-1)[(s-1)^2+4]} + \frac{3(s-1)}{s(s-1)}$$

$$= \frac{5}{s[(s-1)^2+4]} + \frac{3}{s}$$

3. Partial Fractions

$$\frac{5}{s[(s-1)^2+4]} = \frac{A}{s} + \frac{Bs+C}{(s-1)^2+4} = \frac{1}{s} - \frac{s-2}{(s-1)^2+4}$$

$$\Rightarrow 5 = A[(s-1)^2+4] + Bs^2 + Cs$$

$$s=0 \Rightarrow 5 = 5A+0+0 \Rightarrow A=1$$

$s=1 \Rightarrow 5 = 1(4)+B+C \Rightarrow B+C=1$

$s=-1 \Rightarrow 5 = 1(8)+B-C \Rightarrow B-C=-3$

$2B=-2 \Rightarrow B=-1, C=2$

4. Find $y(t)$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3}{s} + \frac{1}{s} - \frac{s-2}{(s-1)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{4}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+4}\right\}$$

$$= 4\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+2^2}\Big|_{s\to s-1}\right\} + \mathcal{L}^{-1}\left\{\left(\frac{2}{2}\right)\left(\frac{1}{s^2+2^2}\right)\Big|_{s\to s-1}\right\}$$

$$= 4 - e^t\cos(2t) + \frac{1}{2}e^t\sin(2t)$$

Bonus (2pts): What other methods can you use to solve this IVP? What would be the potential drawbacks to each method?

MUC -> ton of algebra / Annihilator / Product Rule.

VOP -> 3x3 Wronskian / Trig identities