

MTH 204

Quiz 8

4 April 2008

Name: _____

Section: _____

Follow the directions carefully.
Please show all your work
neatly in pencil. This quiz is
closed book, closed notes, but you
may use your homework
solutions. A copy of the Laplace
transform table is on the back
page. If you get stuck, feel
free to ask me for help.

Solve the integrodifferential equation

$$y'(t) = 1 - \int_0^t e^{-2\tau} y(t-\tau) d\tau$$

1. Take the Laplace of both sides

$$\mathcal{L}\{y'(t)\} = \mathcal{L}\{1\} - \mathcal{L}\{e^{-2t} * y(t)\}$$

$$sY(s) - y(0) = \frac{1}{s} - \mathcal{L}\{e^{-2t}\} \mathcal{L}\{y(t)\}$$

$$sY(s) - 1 = \frac{1}{s} - \left(\frac{1}{s+2}\right)Y(s)$$

2. Solve for $Y(s)$

$$\left(\frac{s+1}{s+2}\right)Y(s) = \frac{1}{s} + 1$$

$$\left(\frac{s(s+2)+1}{s+2}\right)Y(s) = \frac{1}{s} + 1 = \frac{s+1}{s}$$

$$Y(s) = \left(\frac{s+1}{s}\right)\left(\frac{s+2}{(s+1)^2}\right) = \frac{s+2}{s(s+1)}$$

3. Partial Fractions

$$\begin{aligned} \frac{s+2}{s(s+1)} &= \frac{A}{s} + \frac{B}{s+1} \Rightarrow s+2 = A(s+1) + Bs \\ s=0 &\Rightarrow 2=A \\ s=-1 &\Rightarrow 1=-B \\ &= \frac{2}{s} - \frac{1}{s+1} \end{aligned}$$

4. Take the inverse Laplace of both sides

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = 2 - e^{-t}$$