MTH 204 Name Key
Quiz 9 Section: A & C
7 Nov 2008

Please follow the directions carefully. You must show all your work neatly in pencil to receive full credit. This quiz is closed book, closed notes, but you may use your homework solutions. A copy of the Laplace transform table is on the back page. If you get stuck, feel free to ask me for help.

LEAD: Thurs 5-7 PM
CSF G5D

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Solve the integro-differential equation

$$y'(t) - 6y(t) + 9\int_0^t y(t-\tau)\,d\tau = 1, \quad y(0) = 2$$

$$y'(t) - 6y(t) + 9(1 * y(t)) = 1$$

1. Take the Laplace of both sides

$$\mathcal{L}\{y'(t)\} - 6\mathcal{L}\{y(t)\} + 9\mathcal{L}\{1 * y(t)\} = \mathcal{L}\{1\}$$

$$(sY(s) - \underbrace{y(0)}_{2}) - 6Y(s) + \frac{9}{s}Y(s) = \frac{1}{s}$$

$$\left(s - 6 + \frac{9}{s}\right)Y(s) = \frac{1}{s} + 2$$

2. Solve for $Y(s)$

$$(s^2 - 6s + 9)Y(s) = 1 + 2s$$

$$\Rightarrow Y(s) = \frac{1}{(s-3)^2} + \frac{2s}{(s-3)^2} = \frac{1}{(s-3)^2} + \frac{2(s-3+3)}{(s-3)^2}$$

$$= \frac{7}{(s-3)^2} + \frac{2}{s-3}$$

3. Find $y(t)$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= 7\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$= 7te^{3t} + 2e^{3t}$$

Bonus (1pt each) The Elementary row operations are