MTH 204
Quiz 10
14 Nov 2008

Name: Key
Section: A&C

Follow the directions carefully. You must show all your work to get full credit. Please write neatly in pencil. This quiz is closed book, closed notes, but you can use your homework solutions. If you get stuck, feel free to ask me for help

LEAD - Thursdays
5:00-7:00 PM
CSF G5D.

1. Consider $\vec{w}(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$ and $\vec{z}(t) = \begin{bmatrix} t-1 \\ t \end{bmatrix} e^{2t}$. Do $\vec{w}(t)$ and $\vec{z}(t)$ form a FSS for $\vec{x}' = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \vec{x}$. State why or why not.

Note: For $\vec{w}$ and $\vec{z}$ to form a FSS,

1. $\vec{w}, \vec{z}$ must sols
2. $\vec{w}, \vec{z}$ must be LI
3. # LI sol = # cols A

LHS: $\vec{w}' = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$

RHS: $A\vec{w} = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} = \begin{bmatrix} -1+3 \\ -3+5 \end{bmatrix} e^{2t} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{2t}$

$\Rightarrow \vec{w}$ is a sol

LHS: $z' = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + 2\begin{bmatrix} t-1 \\ t \end{bmatrix} e^{2t} = \begin{bmatrix} 2t-1 \\ 2t+1 \end{bmatrix} e^{2t}$

RHS: $A\vec{z} = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} t-1 \\ t \end{bmatrix} e^{2t} = \begin{bmatrix} -t+1+3t \\ -3t+3+5t \end{bmatrix} e^{2t}$

$= \begin{bmatrix} 2t+1 \\ 2t+3 \end{bmatrix} e^{2t}$

$\Rightarrow \vec{z}$ is not a sol.

$\vec{w}$ and $\vec{z}$ do not form a FSS since $\vec{z}$ is not a sol.

2. Solve $\vec{x}' = \begin{bmatrix} -4 & 3 \\ 3 & 4 \end{bmatrix} \vec{x}$

1. Find λ's

$0 = \det(A - \lambda I) = \lambda^2 - \mathrm{Tr}A\lambda + \det A = \lambda^2 - 0\lambda - 25$

$\Rightarrow \lambda = 5, -5$

2. Find $\vec{K}$'s

For $\lambda_1 = 5$, $(A - \lambda_1 I)\vec{K}_1 = \vec{0}$

$$\begin{bmatrix} -4-5 & 3 \\ 3 & 4-5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} -9 & 3 & 0 \\ 3 & -1 & 0 \end{array}\right] \Rightarrow R_1 = -3R_2$$

$3u_1 - u_2 = 0 \qquad \Rightarrow \vec{K}_1 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ 3u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$u_2 = 3u_1$

$\hookrightarrow FV = 1 \quad \Rightarrow \vec{x}_1(t) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{5t}$

For $\lambda_2 = -5$, $(A - \lambda_2 I)\vec{K}_2 = \vec{0}$

$$\begin{bmatrix} -4-(-5) & 3 \\ 3 & 4-(-5) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 3 & 9 & 0 \end{array}\right] \Rightarrow R_2 = 3R_1$$

$v_1 + 3v_2 = 0 \qquad \Rightarrow \vec{K}_2 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3v_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$v_1 = -3v_2$

$\hookrightarrow FV = 1 \quad \Rightarrow \vec{x}_2(t) = \begin{bmatrix} -3 \\ 1 \end{bmatrix} e^{-5t}$

3. Find general solution

$\vec{x}(t) = c_1\vec{x}_1 + c_2\vec{x}_2$

$= c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix} e^{-5t}$