MTH 204
Quiz 10
18 April 2008

Name Key
Section C & F

Follow the directions carefully. Please show all your work neatly in pencil. You must calculate the eigenvalues and eigenvectors by hand. This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

Solve $\vec{x}' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}\vec{x}$

1. Find $\lambda$

$0 = \det(A-\lambda I) = \lambda^2 - (1+3)\lambda + 4 = \lambda^2 + 4\lambda + 4 = (\lambda+2)^2$

$\Rightarrow \lambda = 2, 2.$

2. Find $\vec{K}$

For $\lambda = 2$, $(A-\lambda I)\vec{K} = \vec{0}$

$\left[\begin{array}{cc|c} 1-2 & -1 & 0 \\ 1 & 3-2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right] \Rightarrow u_1 + u_2 = 0$

$u_1 = -u_2$ ← FV=1

$\Rightarrow \vec{K} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -u_2 \\ u_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\Rightarrow \vec{x}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t}$

3. Find $\vec{P}$

For $\lambda = 2$, $(A-\lambda I)\vec{P} = \vec{K}$

$\left[\begin{array}{cc|c} 1-2 & -1 & -1 \\ 1 & 3-2 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} -1 & -1 & -1 \\ 1 & 1 & 1 \end{array}\right] \Rightarrow p_1 + p_2 = 1$

$p_1 = -p_2 + 1$

↳ FV=0

$\Rightarrow \vec{P} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -p_2+1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow \vec{x}_2 = \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) e^{2t}$

4. GS: $\vec{x}(t) = c_1\vec{x}_1 + c_2\vec{x}_2$

$= c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) e^{2t}$