MTH 204
Quiz 10
17 Apr 2009

Name: Key

Section: C or F (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Exam 3: Friday, 24 Apr
7.4 - 8.3

Solve the IVP $\vec{x}' = \begin{bmatrix} 5 & 1 \\ -4 & 1 \end{bmatrix}\vec{x}$, $\vec{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

1. Find λ

$0 = \det(A-\lambda I) = \lambda^2 - \mathrm{Tr}A\lambda + \det A = \lambda^2 - 6\lambda + 9 = (\lambda-3)^2$

$\Rightarrow \lambda = 3, 3$

2. Find $\vec{K}$

For $\lambda = 3$, $(A-\lambda I)\vec{K} = \vec{0}$

$\begin{bmatrix} 5-3 & 1 \\ -4 & 1-3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 0 \\ \cancel{-4} & \cancel{-2} & \cancel{0} \end{array}\right]$ $\quad R_2 = -2R_1$

$2u_1 + u_2 = 0$

$u_2 = -2u_1 \rightarrow FV = 1$

$\Rightarrow \vec{K} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ -2u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\Rightarrow \vec{x}_1(t) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{3t}$

3. Find $\vec{P}$

For $\lambda = 3$, $(A-\lambda I)\vec{P} = \vec{K}$

$\begin{bmatrix} 5-3 & 1 \\ -4 & 1-3 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ \cancel{-4} & \cancel{-2} & \cancel{-2} \end{array}\right]$ $\quad R_2 = -2R_1$

$2p_1 + p_2 = 1$

$p_2 = 1 - 2p_1 \leftarrow FV = 0$

$\Rightarrow \vec{P} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ -2p_1 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\Rightarrow \vec{x}_2(t) = \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) e^{3t}$

4. GS: $\vec{x}(t) = c_1\vec{x}_1(t) + c_2\vec{x}_2(t) = c_1\begin{bmatrix} 1 \\ -2 \end{bmatrix}e^{3t} + c_2\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}t + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)e^{3t}$

5. IC $\vec{x}(0) = c_1\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\quad \Rightarrow c_1 = 2$

$\Rightarrow -2c_1 + c_2 = -1$

$\Rightarrow c_2 = 3$

So $\vec{x}(t) = 2\vec{x}_1(t) + 3\vec{x}_2(t)$