MTH 204
Quiz 11
21 Nov 2008

Name Key
Section A & C

Follow the directions carefully. Please write neatly in pencil. You must show all your work to receive full credit. This quiz is closed book, closed notes, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

LEAD - Thursdays
CSF G5D, 5-7 PM

Exam 3 - 5 Dec
7.4 - 8.3

Solve the IVP $\vec{x}' = \begin{bmatrix} 1 & -8 \\ 1 & -3 \end{bmatrix} \vec{x}$, $\vec{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

1. Find the λ's

$0 = \det(A - \lambda I) = \lambda^2 - \mathrm{Tr}A\lambda + \det A = \lambda^2 + 2\lambda + 5$

$\Rightarrow \lambda = \dfrac{-2 \pm \sqrt{4 - 4(1)5}}{2(1)} = \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$

Use $\lambda = -1 + 2i$

2. Find $\vec{K}$.

For $\lambda = -1 + 2i$, $(A - \lambda I)\vec{K} = \vec{0}$

$$\begin{bmatrix} 1-(-1+2i) & -8 \\ 1 & -3-(-1+2i) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2(1-i) & -8 & 0 \\ 1 & -2(1+i) & 0 \end{array}\right] \qquad R_2 = \frac{(1+i)}{4} R_1$$

$2(1-i)u_1 - 8u_2 = 0 \Rightarrow u_2 = \dfrac{2(1-i)}{8} u_1 = \dfrac{(1-i)}{4} u_1$ ↳ FV=4

$$\Rightarrow \vec{K} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ \frac{(1-i)u_1}{4} \end{bmatrix} = \begin{bmatrix} 4 \\ 1-i \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

3. Find $\vec{z}$

$\vec{z}(t) = \vec{K} e^{\lambda t}$

$= \vec{K} e^{(-1+2i)t} = \vec{K} e^{-t} e^{i2t} = \vec{K} e^{-t}(\cos(2t) + i\sin(2t))$

$= \left(\begin{bmatrix} 4 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right) e^{-t}(\cos(2t) + i\sin(2t))$

$= \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{-t}\cos(2t) + i \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{-t}\sin(2t) + i \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-t}\cos(2t)$

$- \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-t}\sin(2t)$.

4. Find $\vec{x}_1$ and $\vec{x}_2$

$$\vec{x}_1(t) = \mathrm{Re}\,\vec{z} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{-t}\cos(2t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t}\sin(2t).$$

$$= \begin{bmatrix} 4\cos(2t) \\ \cos(2t)+\sin(2t) \end{bmatrix} e^{-t}$$

$$\vec{x}_2(t) = \mathrm{Im}\,\vec{z} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{-t}\sin(2t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-t}\cos(2t)$$

$$= \begin{bmatrix} 4\sin(2t) \\ \sin(2t)-\cos(2t) \end{bmatrix} e^{-t}$$

GS: $\vec{x}(t) = c_1\vec{x}_1(t) + c_2\vec{x}_2(t)$

5. IC

$$\vec{x}(0) = c_1\begin{bmatrix} 4 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \begin{array}{l} 4c_1 = 2 \Rightarrow c_1 = 1/2 \\ c_1 - c_2 = 3 \Rightarrow c_2 = -5/2 \end{array}$$

$$\Rightarrow \vec{x}(t) = \frac{1}{2}\vec{x}_1(t) - \frac{5}{2}\vec{x}_2(t)$$