

MTH204

Quiz 11

2 May 2008

Name Key

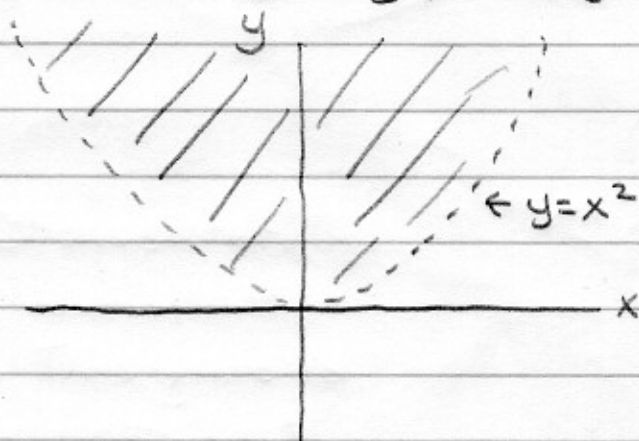
Section C & F

Follow the directions carefully.
Please show all your work
neatly in pencil. This quiz is
closed book, closed notes, but
you may use your home work
solutions. If you get stuck,
feel free to ask me for help.

1. Find and sketch a region in the xy -plane for which $y' = f(x, y) = \sqrt{y - x^2}$ is guaranteed to have a unique solution whose graph passes through any pt (x_0, y_0) in the region.

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{y-x^2}}$$

$$R = \{(x, y) : y - x^2 > 0\} \\ = \{(x, y) : y > x^2\}$$



2. Find the general solution to $xy' - y = x^2 \sin(x)$ on $(0, \infty)$.

1. Put DE in std form: $y' - x^{-1}y = x \sin(x)$

2. Find IF

$$IF = e^{-\int x^{-1} dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1}$$

3. Multiply both sides by IF

$$x^{-1}(y' - x^{-1}y) = x^{-1}(x \sin(x))$$

$$x^{-1}y' - x^{-2}y = \sin(x)$$

4. Rewrite LHS as product rule

$$\frac{d}{dx} [x^{-1}y] = \sin(x)$$

5. Integrate wrt x

$$\int \frac{d}{dx} [x^{-1}y] dx = \int \sin(x) dx$$

$$x^{-1}y = -\cos(x) + C$$

$$\Rightarrow y(x) = -x \cos(x) + Cx$$