MTH 204
Quiz 1.
1 Sept 2005.

Name: Key
Section: A & C.

Read the directions carefully.
You must show all your work
to get partial credit.

Closed book, closed notes.

1. State the order of the given ODE. Determine whether the equation is linear or nonlinear. If the equation is nonlinear, state why.

a. $t^5 y^{(4)} - t^3 y''' + 6y = 0$

4th order

Linear

b. $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$

2nd order

Nonlinear - degree on $\frac{dy}{dx}$ is not 1.

c. $(1-x)y'' - 4xy' + 5y = \cos 4x$

2nd order

Linear.

2. Determine a region in the xy-plane for which $(4-y^2)y' = x^2$ would have a unique solution whose graph passes through (x_0, y_0) in the region.

$\frac{dy}{dx} = f(x,y) = \frac{x^2}{4-y^2}$ x cont on $\mathbb{R}$; y cont everywhere except $-2, 2$

$\frac{\partial f(x,y)}{\partial y} = \frac{(4-y^2)(x^2)' - (x^2)(4-y^2)'}{(4-y^2)^2}$

$= \frac{-2x^2y}{(4-y^2)^2}$ x cont on $\mathbb{R}$; y cont everywhere except $-2, 2$

Region: x cont on $\mathbb{R}$

y cont everywhere except $-2, 2$.

3. Find all the critical points for $\frac{dy}{dx} = y^2(y^2-4)$ and classify each as either asymptotically stable, semi-stable, or unstable. Draw the phase plane and sketch the typical solution curves in the xy-plane determined by the graphs of the equilibrium solutions.

$y^2(y^2-4)=0$

$y=0 \quad y^2-4=0$

$y=\pm 2.$

Int	TV	Sign	Conclusion
$(-\infty,-2)$	-3	$+$	$\uparrow$
$(-2,0)$	-1	$-$	$\downarrow$
$(0,2)$	1	$-$	$\downarrow$
$(2,\infty)$	3	$+$	$\uparrow$