MTH 204
Quiz 1
1 Sept 2006

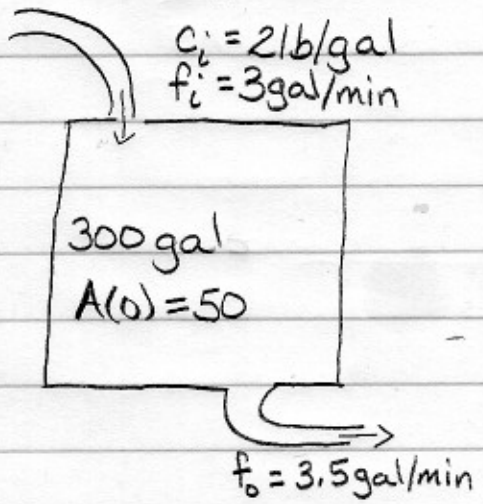
Name: Key
Section B

Please write neatly in pencil. You must show all your work in order to get full credit. You may use your homework solutions, but not your books and notes. If you have any trouble, please feel free to ask me for help.

Test 1: 8 Sept
1.1 - 2.3

LEAD: Thursdays 5-7PM
CSF G5D

1. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt has been dissolved. Another brine solution is pumped into the tank at a rate of 3gal/min, and then when the solution is well stirred, it is pumped out at a rate of 3.5gal/min. If the concentration of the solution entering is 2lb/gal, determine an initial value problem for the amount A(t) of salt at time t.



$$\frac{dA}{dt} = R_i - R_o$$

R_i = (concentration)(flow)
= (2 lb/gal)(3 gal/min)
= 6 lb/min

R_o = (concentration)(flow)
$= \left(\frac{\text{amount}}{\text{volume}}\right)(\text{flow})$
$= \left(\frac{A(t)}{300+(3-3.5)t}\right)(3.5\ \text{gal/min})$

$$\Rightarrow \frac{dA}{dt} = 6 - \frac{\frac{7}{2}A}{300-\frac{1}{2}t} = 6 - \frac{7A}{600-t}$$

$$A(0) = 50$$

2. Consider the differential equation $\frac{dy}{dx} = y^2(4-y^2)$

a. Find the order of the DE. Determine whether or not the equation is (non)linear and (non)autonomous.

1st order

nonlinear

autonomous

b. Find and classify each of the equation's critical point. Draw the corresponding phase line.

$\frac{dy}{dx} = y^2(4-y^2) = 0$

$\Rightarrow$ Critical points: $y = -2, 0, 2$

Interval	Test Value	+/−	↑/↓
$(-\infty, -2)$	-3	−	↓
$(-2, 0)$	-1	+	↑
$(0, 2)$	1	+	↑
$(2, \infty)$	3	−	↓