MTH 204
Quiz 2
16 Sept 2005

Name _____
Section _____

Follow the directions carefully.
Show all of your clearly in pencil.

1. Find the general solution to the following DE.

$$x\frac{dy}{dx} - y = x^2\sin(x).$$

Find the largest interval I over which the general solution is defined.

$$\frac{dy}{dx} - \frac{y}{x} = x\sin(x)$$

$$\text{IF: } e^{-\int \frac{1}{x}dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1}$$

$$x^{-1}\left(\frac{dy}{dx} - \frac{y}{x} = x\sin(x)\right)$$

$$x^{-1}\frac{dy}{dx} - \frac{y}{x^2} = \sin(x)$$

$$\frac{d}{dx}\left[x^{-1}y\right] = \sin(x)$$

$$\int \frac{d}{dx}\left[x^{-1}y\right]dx = \int \sin(x)dx$$

$$x^{-1}y = -\cos(x) + C$$

$$y(x) = Cx - x\cos(x)$$

$$x\frac{dy}{dx} - y = x^2\sin(x)$$

$$\frac{dy}{dx} - \underbrace{\frac{1}{x}}_{P(x)}y = \underbrace{x\sin(x)}_{Q(x)}$$

$P(x)$: $(-\infty, 0) \cup (0, \infty)$

$Q(x)$: $\mathbb{R}$

I: $x > 0$

2. Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose 10 mins later, the temperature of the beer is 48°F. Use Newton's law of cooling to find the temperature 25 mins after the beer was placed in the room.

$$\begin{cases} \frac{dT}{dt} = k(T - T_m) = k(T-70) \\ T(0) = 40\,^\circ F \\ T(10) = 48\,^\circ F \end{cases}$$

$$\int \frac{dT}{T-70} = \int k\,dt$$

$$\ln|T-70| = kt + C$$

$$T - 70 = c_1 e^{kt}$$

$$T(t) = 70 + c_1 e^{kt}$$

$$T(0) = 40 = 70 + c_1$$

$$\Rightarrow c_1 = -30$$

$$T(t) = 70 - 30e^{kt}$$

$$T(10) = 48 = 70 - 30e^{10k}$$

$$-30e^{10k} = -22$$

$$e^{10k} = \frac{11}{15}$$

$$k = \frac{1}{10}\ln\left(\frac{11}{15}\right) \approx -.031$$

So $T(t) = 70 - 30e^{-.031t}$

Then $T(25) = 70 - 30e^{-.031(25)}$

$$\approx 56.18\,^\circ F$$