MTH 204
Quiz 2
15 Sept 2006

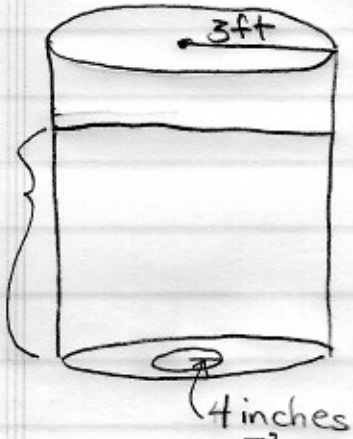
Name: Key
Section B.

Follow the directions carefully. Please write neatly in pencil and show all your work in order to get full credit. DO NOT use decimals on any intermediate step. If you get stuck, feel free to ask me for help.

LEAD: Thursdays 5:00 - 7:00
CSF G5D.

A tank in the form of a right circular cylinder standing on end is leaking water through a circular hole in the bottom. When friction and contraction of water at the hole are ignored, the height h of the water in the tank is described by $\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2gh}$

where A_w and A_h are the cross-sectional areas of the water and the hole, respectively. Now suppose that the tank is 20 feet high and has a radius of 3 feet with a circular hole with radius 4 inches. If the tank is initially three quarters full, how long will it take to empty? Assume $g = 32\ ft/s^2$.



$h(0) = \frac{3}{4}(20) = 15$

$A_h = \pi r^2 = \pi\left(\frac{4}{12}\right)^2 = \frac{\pi^2}{9}$

$A_w = \pi r^2 = \pi(3)^2 = 9\pi$

$$\Rightarrow \frac{dh}{dt} = -\frac{\pi^2/9}{9\pi^2}\sqrt{2gh} = \frac{-8\sqrt{h}}{81}$$

$h(0) = 15$

Note: This is a 1st order, nonlinear, separable, autonomous equation

$$\Rightarrow \int \frac{dh}{\sqrt{h}} = \frac{-8}{81}\int dt$$

$$2\sqrt{h} = -\frac{8}{81}t + C$$

$\sqrt{h} = -\frac{4}{81}t + c_1 \quad$ where $c_1 = \frac{1}{2}c$

$h(t) = \left(-\frac{4}{81}t + c_1\right)^2$

$h(0) = c_1^2 = 15 \quad \Rightarrow c_1 = \sqrt{15}$

So $h(t) = \left(-\frac{4}{81}t + \sqrt{15}\right)^2$

Then let t_1 be the time the tank is completely emptied.

$h(t_1) = \left(-\frac{4}{81}t_1 + \sqrt{15}\right)^2 = 0$

$\Rightarrow -\frac{4}{81}t_1 + \sqrt{15} = 0$

$\Rightarrow t_1 = \frac{81\sqrt{15}}{4}$

$\approx 78.43s$