MTH204 Name Key

Quiz 2 Section A/C

5 Sept 2008

Follow the directions carefully. Please show all your work neatly in pencil. This quiz is closed notes, closed book, but you may use your homework solutions. If you get stuck, feel free to ask me for help.

LEAD - Thursdays
CSF G5D
5:00 - 7:00 PM

Consider the DE $y(x-2)\frac{dy}{dx} = \frac{2x\sec(y)}{x+3}$.

(a). Give the order of the DE. Determine if the DE is (non)linear, (non)autonomous, and (non)separable. State why.

1st order. Nonlinear since we have a nonlinear function of y. Nonautonomous since x shows up. Separable since DE can be rewritten as $y' = f(x,y) = g(x)h(y)$.

(b). Solve the DE. Methods { SoV → separable; ~~IF~~ → nonlinear.

$$\Rightarrow \int y\cos(y)\,dy = \int \frac{2x}{(x-2)(x+3)}\,dx$$

LHS: $\int y\cos(y)\,dy = y\sin(y) - \int \sin(y)\,dy$ $\quad u=y \quad dv=\cos(y)$

$= y\sin(y) + \cos(y) + C_1$ $\quad du=dy \quad v=\sin(y)$

RHS: $\frac{2x}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{4/5}{x-2} + \frac{6/5}{x+3}$

$\Rightarrow 2x = A(x+3) + B(x-2)$

$x=2 \Rightarrow 4 = 5A \Rightarrow A = 4/5$

$x=-3 \Rightarrow -6 = -5B \Rightarrow B = 6/5$

$$\Rightarrow \int\left(\frac{4/5}{x-2} + \frac{6/5}{x+3}\right)dx = \frac{4}{5}\ln|x-2| + \frac{6}{5}\ln|x+3| + C_2$$

So $y\sin(y) + \cos(y) = \frac{4}{5}\ln|x-2| + \frac{6}{5}\ln|x+3| + C$