MTH 204
Quiz 2
5 Feb 2006

Name: Key
Section: D\E

READ the directions carefully
Write neatly in pencil and show all your work
Do not use decimals
If you have trouble, feel free to ask me for help.

LEAD Session: Wednesdays
6:00-7:30 PM
CSF G5D

A large tank is partially filled with 100 gals of fluid in which 10 pounds of salt is dissolved. Brine containing $\frac{1}{2}$ pound of salt per gallon is pumped into the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a rate of 4 gal/min.

a. Set up the IVP and classify the DE by order, linearity, separability, and state whether or not the equation is autonomous

b. Find the number of pounds of salt in the tank after 30 mins.

a. Rate in: (flow)(concentration) $= (6\text{gal/min})(\frac{1}{2}\text{lb/gal})$
$= 3\text{ lb/min}.$

Rate out: $\left(\frac{4\text{gal}}{\text{min}}\right)\left(\frac{A(t)}{100+(6-4)t}\right) = \frac{2A}{50+t}$

Then $\frac{dA}{dt} = R_i - R_o \Rightarrow \frac{dA}{dt} = 3 - \frac{2A}{50+t}$

$A(0) = 10.$

1st order, linear, nonseparable, not autonomous

b. $\frac{dA}{dt} + \underbrace{\left(\frac{2}{50+t}\right)}_{P(t)}A = 3$

$IF: e^{\int P(t)dt} = e^{2\int \frac{dt}{50+t}} = e^{2\ln|50+t|} = e^{\ln|50+t|^2} = (50+t)^2$

$$(50+t)^2\left[\frac{dA}{dt} + \left(\frac{2}{50+t}\right)A = 3\right]$$

$$(50+t)^2\frac{dA}{dt} + 2(50+t)A = 3(50+t)^2$$

$$\frac{d}{dt}\left[(50+t)^2 A\right] = 3(50+t)^2$$

$$\int \frac{d}{dt}\left[(50+t)^2 A\right]dt = 3\int (50+t)^2 dt$$

$$(50+t)^2 A = (50+t)^3 + C$$

$$A(t) = 50+t + \frac{C}{(50+t)^2}$$

$$A(0) = 10 = 50 + \frac{C}{50^2}$$

$$\Rightarrow C = -100{,}000$$

$$\text{So } A(t) = 50+t - \frac{100{,}000}{(50+t)^2}$$

$$\text{Then } A(30) = 50+30 - \frac{100{,}000}{(50+30)^2}$$

$$\approx 64.38 \text{ lb.}$$