MTH 204
Quiz 2
2 Feb 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
If you get stuck, feel free to ask me for help.

LEAD: Thursdays, 5:00-7:00
CSF G5D

A model for the population P(t) in a suburb of a large city is given by the initial value problem

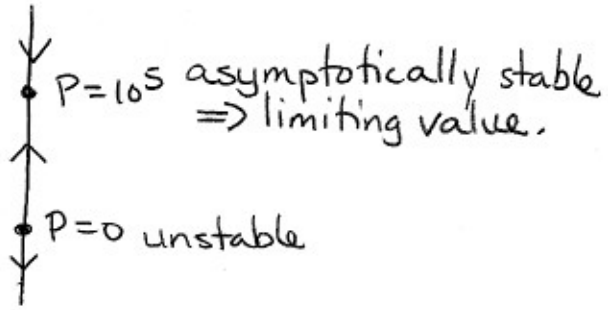
$$\frac{dP}{dt} = P(10^{-1} - 10^{-6}P), \quad P(0) = 4000,$$

where t is measured in months. What is the limiting value of the population? At what time will the population be equal to one-half of this limiting value?

$\frac{dP}{dt} = 10^{-6}P(10^5 - P) = 0$

CP: $P = 0, 10^5$

Int	TV	+/-	↑/↓
$(-\infty, 0)$	-10^5	−	↓
$(0, 10^5)$	10	+	↑
$(10^5, \infty)$	10^6	−	↓



$\Rightarrow \frac{10^6 dP}{P(10^5 - P)} = dt$

$\frac{10^6}{P(10^5-P)} = \frac{A}{P} + \frac{B}{10^5 - P} = \frac{10}{P} + \frac{10}{10^5 - P}$

$10^6 = A(10^5 - P) + BP$

$P = 0 \Rightarrow 10^5 A = 10^6 \Rightarrow A = 10$

$P = 10^5 \Rightarrow 10^5 B = 10^6 \Rightarrow B = 10$

$10\int\left(\frac{1}{P} + \frac{1}{10^5 - P}\right)dP = \int dt$

$10[\ln|P| - \ln|10^5 - P|] = t + c$

$10 \ln\left|\frac{P}{10^5 - P}\right| = t + c$

$10 \ln\left|\frac{4000}{10^5 - 4000}\right| = 0 + c \Rightarrow c = 10\ln\left|\frac{1}{24}\right|$

$10 \ln\left|\frac{P}{10^5 - P}\right| = t + 10\ln\left|\frac{1}{24}\right|$

Let $P(t_1) = \frac{10^5}{2}$

$t_1 + 10\ln\left|\frac{1}{24}\right| = 10 \ln\left|\frac{10^5/2}{10^5 - \frac{10^5}{2}}\right| = 10\ln|1| = 0$

$\Rightarrow t_1 = -10\ln\left|\frac{1}{24}\right| = 10\ln|24| \approx 31.78$ months

Bonus (2pts): Who will win the Super Bowl, the Colts or the Bears?

Colts 29, Bears 17