MTH 204.
Quiz 3
23 Sept 2005

Name: Key

Section: A/C

Follow directions carefully. Show <u>all</u> your work neatly and clearly in pen. <u>Do not</u> convert to decimals in any intermediate step.

<u>Note</u>: Test 2 is 30 Sept

1. A tank in the form of a right circular cylinder standing on end is leaking water through a circular hole in its bottom. When friction and contraction of water at the hole are ignored, the height h of water in the tank is described by

$$\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2gh}$$

where A_w and A_h are the cross-sectional areas of the water and the hole, respectively. Suppose the tank is 16ft high and has a radius of 3ft. Suppose the radius of the circular hole is 1in. If the tank is initially full, how long will it take to emty? Use $g = 32 ft/s^2$

$$A_h = \pi\left(\frac{1}{12}\right)^2 = \frac{\pi}{144}$$

$$A_w = \pi(3)^2 = 9\pi$$

$$\Longrightarrow \begin{cases} \dfrac{dh}{dt} = \dfrac{-\frac{\pi}{144}}{9\pi}\sqrt{2(32)h} \\ h(0) = 16 \end{cases}$$

$$\frac{dh}{dt} = \frac{-\cancel{\pi}}{144(9\cancel{\pi})}8\sqrt{h} = -\frac{\sqrt{h}}{162}$$

$$\int\frac{dh}{\sqrt{h}} = \int\frac{-1}{162}dt$$

$$2\sqrt{h} = -\frac{t}{162} + c$$

$$\sqrt{h} = \frac{-t}{324} + c_1$$

$$h(t) = \left(\frac{-t}{324} + c_1\right)^2$$

$$h(0) = 16 = \left(\frac{-0}{324} + c_1\right)^2$$

$$c_1^2 = 16 \Rightarrow c_1 = 4$$

$$\text{So } h(t) = \left(\frac{-t}{324} + 4\right)^2$$

$$0 = \left(\frac{-t}{324} + 4\right)^2$$

$$0 = \frac{-t}{324} + 4$$

$$\frac{t}{324} = 4$$

$$t = 4(324) = 1296\,s$$

$$\approx 21.6\,mins.$$

2. Determine whether or not the functions $f_1(x)=x^2$, $f_2(x)=x^3$ are linearly independent on $(0,\infty)$. Then given the DE $x^2y''-6xy'+12y=0$, check whether or not $f_1(x)$ and $f_2(x)$ form a fundamental set of solutions for the DE. Justify your answer.

$$W(f_1,f_2)(x) = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4 \neq 0 \text{ since } x\in(0,\infty)$$

So $f_1(x)$ and $f_2(x)$ are linearly independent.

For $f_1(x)=x^2$, $f_1'(x)=2x$, $f_1''(x)=2$

LHS: $x^2(2)-6x(2x)+12x^2 = 2x^2-12x^2+12x^2 = 2x^2$

RHS: 0

so x^2 is not a solution.

For $f_2(x)=x^3$, $f_2'(x)=3x^2$, $f_2''(x)=6x$.

RHS: 0

LHS: $x^2(6x)-6x(3x^2)+12x^3 = 6x^3-18x^3+12x^3=0$

So x^3 is a solution.

While $f_1(x)$ and $f_2(x)$ are linearly independent, only $f_2(x)$ is a solution to the DE. So $f_1(x)$ and $f_2(x)$ do not form a FSS.