MTH 204
Quiz 3
22 Sept 2006

Name Key
Section B.

Follow the directions carefully. Show <u>all</u> your work neatly in pencil. If you get stuck, feel free to ask *me* for help.

LEAD: Thursdays 5-7 PM
CSF G5D

Test 2: 29 Sept
3.2 – 4.5

1. Consider the DE $2x^2y''+5xy'+y=0$ on the interval $(0,\infty)$.

a. Give the conditions needed to have a fundamental set of solutions (FSS) on $(0,\infty)$

1. y_1, y_2 are solutions to the DE
2. y_1, y_2 are linearly independent.
3. # linearly independent solutions = order of the DE

b. Consider the functions $f(x)=x^{-1/2}$, $g(x)=3x^2$, and $h(x)=2x^{-1}$. Do $f(x), g(x)$ form a FSS? Why or why not? Do $f(x), h(x)$ form a FSS? Why or why not?

$2x^2f''+5xf'+f=2x^2\left(\frac{3}{4}x^{-5/2}\right)+5x\left(-\frac{1}{2}x^{-3/2}\right)+x^{-1/2}=0 \Rightarrow f$ is a solution

$2x^2g''+5xg'+g=2x^2(6)+5x(6x)+3x^2\neq 0 \Rightarrow g$ is not a solution

$2x^2h''+5xh'+h=2x^2(4x^{-3})+5x(-2x^{-2})+2x^{-1}=0 \Rightarrow h$ is a solution

Then $f(x), g(x)$ don't form a FSS since $g(x)$ is not a solution.

$$W(f(x),h(x))=\begin{vmatrix} x^{-1/2} & 2x^{-1} \\ -\frac{1}{2}x^{-3/2} & -2x^{-2}\end{vmatrix}=x^{-1/2}(-2x^{-2})-\left(-\frac{1}{2}x^{-1/2}\right)(2x^{-1})=-x^{-3/2}\neq 0$$

$\Rightarrow f(x), h(x)$ are linearly independent.

Then since $f(x), h(x)$ are linearly independent solutions to the DE, they form a FSS.

2. Find a second linearly independent for $x^2y''+2xy'-6y=0$ on the interval $(0,\infty)$ when $y_1(x)=x^2$. **Do NOT** use the integral equation (no points will be awarded if you do).

$y_2(x)=u(x)y_1(x)=ux^2$

$y_2'=u'x^2+2ux$

$y_2''=u''x^2+2u'x+2u'x+2u=u''x^2+4u'x+2u$

Plugging y_2 into the DE,

$x^2(u''x^2+4u'x+2u)+2x(u'x^2+2ux)-6ux^2=0$

$u''x^4+u'(4x^3+2x^3)+u(2x^2+4x^2-6x^2)=0$

$u''x^4+6u'x^3=0$

$u''+6x^{-1}u'=0$

Let $\begin{cases} w=u' \\ w'=u'' \end{cases}$

$\Rightarrow \dfrac{dw}{dx}=-\dfrac{6w}{x}$

$\displaystyle\int\frac{dw}{w}=-6\int\frac{dx}{x}$

$\ln|w|=-6\ln|x|=\ln|x^{-6}|$

$\Rightarrow w=x^{-6}=u'$

$\Rightarrow u(x)=\int x^{-6}dx=-\frac{1}{6}x^{-5}$

$\Rightarrow u(x)=x^{-5}$

Then $y_2(x)=u(x)y_1(x)=x^{-5}x^2=x^{-3}$

Now the general solution is $y(x)=c_1x^2+c_2x^{-3}$

Bonus (3pts): What's the difference between a superposition and a general solution?

A general solution is a superposition, only with the restriction that each solution is LI and where the # of them = order of the DE.