

MTH204

Quiz 3

12 Sept 2008

Name Key

Section A & C

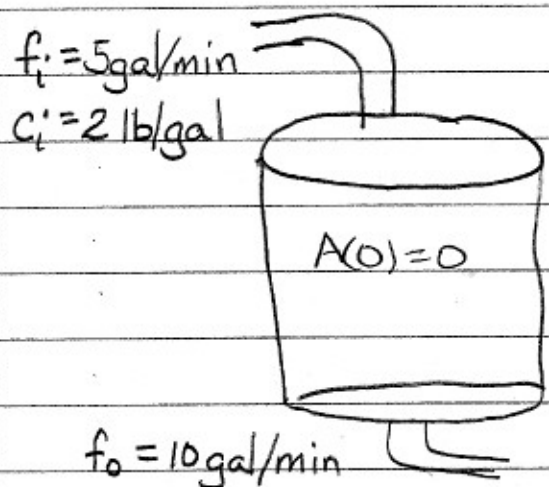
Follow the directions carefully.
Please show all your work neatly
in pencil. This quiz is closed notes,
closed book, but you may use your
homework solutions. If you get
stuck, feel free to ask me for
help.

LEAD - Thursdays

CSF G5D

5:00-7:00PM

Suppose an 800 gal tank is filled with 500 gal of pure water. A brine solution containing 2 lb salt per gal is pumped in at a rate of 5 gal/min. The well mixed solution is then pumped out at a rate of 10 gal/min. (a). Set up the IVP that describes the rate of change in the amount of salt, $A(t)$, at time t



$$\frac{dA}{dt} = R_i - R_o$$

$$= 5(2) - 10 \left(\frac{A}{500 - 5t} \right)$$

$$= 10 - \frac{2A}{100 - t}$$

time	Volume
0	500
1	$500 + 5 - 10 = 500 - 5$
2	$(500 - 5) + 5 - 10 = 500 - 5(2)$
$\vdots$	
t	$500 - 5t$

IVP: $\frac{dA}{dt} = 10 - \frac{2A}{100 - t}$

$A(0) = 0$

(b) Classify the DE. What method(s) do you have to solve this IVP?

1st order, linear, nonautonomous, nonseparable, nonhomogeneous

Methods $\left\{ \begin{array}{l} \text{Sov} \rightarrow \text{not separable} \\ \text{IF} \end{array} \right.$

(c). Solve the IVP.

1. Put DE in std form: $\frac{dA}{dt} + \frac{2}{100-t} A = 10$
 $I: [0, 100)$

$u = 100 - t$
 $du = -dt$

2. Find IF: $IF = e^{\int \frac{2}{100-t} dt} = e^{-2 \ln|100-t|} = (100-t)^{-2}$

3. Multiply both sides by IF.

$$(100-t)^{-2} \left[\frac{dA}{dt} + \frac{2A}{100-t} \right] = 10(100-t)^{-2}$$

$$(100-t)^{-2} \frac{dA}{dt} + \frac{2A}{(100-t)^3} = 10(100-t)^{-2}$$

4. Rewrite LHS as Product Rule of IF times A

$$\frac{d}{dt} [(100-t)^{-2} A] = 10(100-t)^{-2}$$

Note: This only happens when you have the correct function as the IF!

5. Integrate wrt t.

$$\int \frac{d}{dt} [(100-t)^{-2} A] dt = 10 \int (100-t)^{-2} dt$$

$$(100-t)^{-2} A(t) = 10(100-t)^{-1} + C$$

$$A(t) = 10(100-t) + C(100-t)^2$$

6. IC: $A(0) = 0 = 10(100) + C(100)^2 \Rightarrow C = -\frac{1}{10}$

$$\Rightarrow A(t) = 10(100-t) - \frac{1}{10}(100-t)^2$$

Bonus (2pts): After how many minutes will the tank be completely filled/emptied?

The tank is completely empty after 100 minutes.