MTH 204

Quiz 3

10 Feb 2006

Name Key

Section: D/E

Follow the directions carefully.
Please write neatly in pencil and
show all your work.

LEAD Session: Wednesdays
6:00 - 7:30 PM
CSF G5D.

Test 2 : 17 Feb
3.1 - 4.3.

1. Consider the DE $x^2y''-2xy'-4y=0$.
Do the following form a fundamental set of solutions on $(0,\infty)$? If not, explain why. If so, justify your answer.
a. x^{-1}, x^2
b. x^{-1}, x^4
c. $2x^{-1}, x^{-1}$

a. x^{-1}, $(x^{-1})'=-x^{-2}$, $(x^{-1})''=2x^{-3}$
$x^2(2x^{-3})-2x(-x^{-2})-4x^{-1}=2x^{-1}+2x^{-1}-4x^{-1}=0$ x^{-1} is a solution
x^2, $(x^2)'=2x$, $(x^2)''=2$
$x^2(2)-2x(2x)-4x^2=2x^2-4x^2-4x^2=-6x^2\neq 0$ x^2 is not a solution
$\therefore$ x^{-1}, x^2 don't form a FSS since x^2 isn't a solution.

b. x^4, $(x^4)'=4x^3$, $(x^4)''=12x^2$
$x^2(12x^2)-2x(4x^3)-4x^4=12x^4-8x^4-4x^4=0$ x^4 is a solution
$W(x^{-1},x^4)=\begin{vmatrix} x^{-1} & x^4 \\ -x^{-2} & 4x^3 \end{vmatrix}=x^{-1}(4x^3)-x^4(-x^{-2})=4x^2+x^2=5x^2\neq 0$
since $x\in(0,\infty)$
$\Rightarrow x^{-1}, x^4$ are linearly independent.
$\therefore$ Since x^{-1}, x^4 are linearly independent solutions to a 2nd order DE, they form a FSS.

c. While x^{-1}, $2x^{-1}$ are both solutions, they are not linearly indepent, so they do not form a FSS.

2. $y_1(x) = x^2$ is a solution to $x^2y'' - 4xy' + 6y = 0$; $(0, \infty)$.
Use Reduction of Order to find a second linearly independent solution $y_2(x)$. Give the general solution.

Let $y_2(x) = u(x)y_1(x) = ux^2$

$$y_2' = u'x^2 + 2ux$$

$$y_2'' = u''x^2 + 2u'x + 2u'x + 2u = u''x^2 + 4u'x + 2u$$

$$x^2 y_2'' - 4xy_2' + 6y_2 = 0$$

$$x^2[u''x^2 + 4u'x + 2u] - 4x[u'x^2 + 2ux] + 6ux^2 = 0$$

$$u''x^4 + u'[\underbrace{4x^3 - 4x^3}_{0}] + u[\underbrace{2x^2 - 8x^2 + 6x^2}_{0}] = 0$$

$$u''x^4 = 0$$

Let $w = u'$

$w' = u''$

$$x^4 \frac{dw}{dx} = 0$$

Note: this is a 1st order, linear, homogeneous, separable equation

$\int dw = \int 0 dx$ since $x \neq 0$

$w = c_1$

but $w = u'$

so $u = \int c_1 dx = c_1x + c_2$ let $c_1 = 1$, $c_2 = 0$

$\therefore u(x) = x$

Then $y_2(x) = u(x)y_1(x) = x(x^2) = x^3$

Now the general solution is $y(x) = c_1y_1(x) + c_2y_2(x)$

$$= c_1x^2 + c_2x^3$$