

MTH 204
Quiz 3
9 Feb 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.

Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).

You may use your homework solutions.

Please do not share calculators during the quiz.

If you get stuck, feel free to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Exam 2: Friday, 16 February
Sections 3.2 - 4.5

Use reduction of order to find a second linearly independent solution to the differential equation

$$x^2 y'' + 9xy' + 16y = 0, \quad x > 0; \quad y_1(x) = x^{-4}. \quad (\text{NO POINTS for the integration formula})$$

$$y_2(x) = u(x)y_1(x) = ux^{-4}$$

$$y_2' = u'x^{-4} - 4ux^{-5}$$

$$y_2'' = u''x^{-4} - 4u'x^{-5} - 4u'x^{-5} + 20ux^{-6}$$

$$= u''x^{-4} - 8u'x^{-5} + 20ux^{-6}$$

$$x^2(u''x^{-4} - 8u'x^{-5} + 20ux^{-6}) + 9x(u'x^{-4} - 4ux^{-5}) + 16ux^{-4} = 0$$

Regroup in terms of u

$$x^{-2}u'' + (-8x^{-3} + 9x^{-3})u' + (20x^{-4} - 36x^{-4} + 16x^{-4})u = 0$$

$$x^{-2}u'' + x^{-3}u' = 0$$

$$\text{Let } \begin{cases} \omega = u' \\ \omega' = u'' \end{cases}$$

$$x^{-2} \frac{d\omega}{dx} + x^{-3}\omega = 0$$

$$x^{-2} \frac{d\omega}{dx} = -x^{-3}\omega$$

$$\Rightarrow \int \frac{d\omega}{\omega} = -\int x^{-1} dx$$

$$\ln|\omega| = -\ln|x| + C = \ln|x^{-1}| + C$$

$$\Rightarrow \omega = e^{\ln|x^{-1}| + C} = c_1 x^{-1} = u'$$

$$u(x) = \int c_1 x^{-1} dx = c_1 \ln x + c_2$$

$$= \ln x \quad \begin{cases} c_1 = 1 \\ c_2 = 0 \end{cases}$$

$$\text{So } y_2(x) = u(x)y_1(x)$$

$$= x^{-4} \ln x$$