MTH 204
Quiz 3
30 Jan 2009

Name: Key

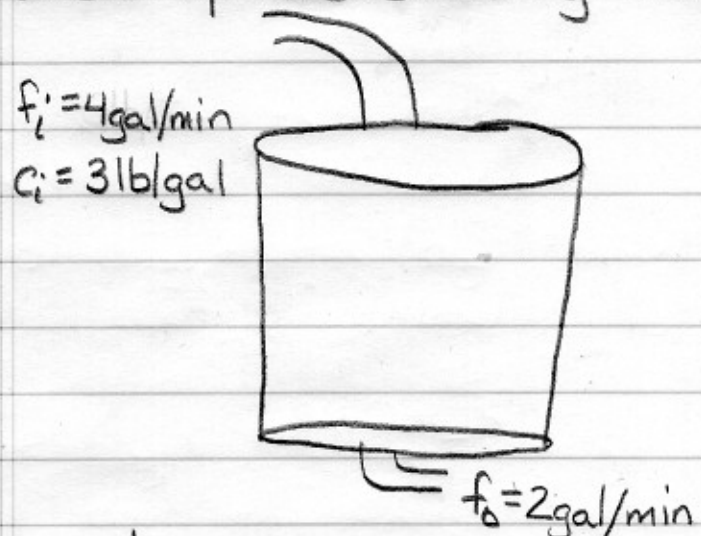
Section: C or F (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

A 800 gal tank contains 400 gal of fluid in which 50 lb of salt is dissolved. Brine containing 3 lb/gal salt is pumped into the tank at a rate of 4 gal/min. The well-mixed solution is then pumped out at a rate of 2 gal/min. Let $A(t)$ = amount of salt in the tank.

a. Set up and classify the IVP



$$\frac{dA}{dt} = R_i - R_o$$
$$= f_i c_i - f_o c_o$$
$$= 4(3) - 2\left(\frac{A}{400+2t}\right)$$
$$= 12 - \frac{A}{200+t}$$

time	Vol
0	400
1	$400+4-2 = 400+2(1)$
⋮	
t	$400+2t$

$A(0) = 50$

1st, linear, nonauto, nonsep, NH

b. What method(s) can you use to solve this IVP?

Methods { ~~SoV~~ nonsep
IF 1st, linear

c. Solve the IVP.

1. Std form: $\frac{dA}{dt} + \frac{A}{200+t} = 12$

2. IF: $e^{\int P(t)dt} = e^{\int \frac{dt}{200+t}} = e^{\ln|200+t|}$

$= (200+t)$

3. $(200+t)\left[\frac{dA}{dt} + \frac{A}{200+t}\right] = 12(200+t)$

$(200+t)\frac{dA}{dt} + A = 12(200+t)$

4. $\frac{d}{dt}\left[(200+t)A\right] = 12(200+t)$

5 $\int \frac{d}{dt}\left[(200+t)A\right]dt = 12\int(200+t)dt$

$(200+t)A = 2400t + 6t^2 + C$

$A(t) = \frac{6t^2 + 2400t + C}{200+t}$

6. $A(0) = 50 = \frac{C}{200} \Rightarrow C = 10{,}000$

$\Rightarrow A(t) = \frac{6t^2 + 2400t + 19{,}000}{200+t}$

d. After how many minutes will the tank be completely drained/full?

$400 + 2t = 800$

$\Rightarrow t = 200$ mins

Bonus (2pts): Who will win the Super Bowl?

Steelers 27

Cardinals 23