MTH 204
Quiz 4
14 Oct 2005

Name Key
Section A\C

Follow the directions carefully. Show all your work neatly in pencil. Do not share calculators. If you have trouble during the quiz, feel free to ask for help.

LEAD Session - Mondays
6PM
Rolla G4

1. Find the <u>smallest</u> differential operator that annihilates the following functions.

2 points each

a. $3e^{-x} + 20e^{2/3x}$
$r=-1 \qquad r=\frac{2}{3}$

$(D+1)(D-\frac{2}{3})[3e^{-x} + 20e^{2/3x}] = 0$

b. $\underbrace{x^4+x^2+3}_{r=0} + 7e^x$
$r=1$

$D^5(D-1)[x^4+x^2+3+7e^x] = 0$

c. $12\cos(2x) + xe^{3x}$
$r=\pm 2i \qquad r=3,3$

$(D^2+4)(D-3)^2[12\cos(2x)+xe^{3x}] = 0$

d. $7x\sin(3x) + 13\cos(3x) + e^{3x}$
$r=\pm 3i, \pm 3i \qquad r=\pm 3i \qquad r=3$

$(D^2+9)^2(D-3)[7x\sin(3x) + 13\cos(3x) + e^{3x}] = 0$

e. $5\cos(x) + 4e^{2x}\cos(2x) + 6x^2.$
$r=\pm i \qquad r=2\pm 2i, \qquad r=0,0,0.$

$(D^2+1)((D-2)^2+4)\,D^3[5\cos(x)+4e^{2x}\cos(2x)+6x^2] = 0$

<u>Hint:</u> What are the three types of annihilators that we looked at?

2. Given the differential equation $3y'' + 12y = \sec(2x)$

a. Classify the differential equation by order, linearity, and state whether or not the equation is homogeneous.

b. What methods are available to solve this equation?

c. Solve the differential equation.

1 $\left\{ \begin{array}{l} \text{a. 2nd order, linear non homogeneous.} \\ \text{b. } g(x) = \sec(2x) \Rightarrow \text{Variation of Parameters} \end{array} \right.$

c. $3y'' + 12y = \sec(2x)$

$y'' + 4y = \frac{1}{3}\sec(2x)$

Solve for y_h.

$y'' + 4y = 0$ Let $y(x) = e^{rx}$

$r^2 + 4 = 0$

$r = \pm 2i$

$y_1 = \cos(2x)$

$y_2 = \sin(2x)$

1 $y_h = c_1\cos(2x) + c_2\sin(2x)$

1 $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix} = 2\cos^2(2x) + 2\sin^2(2x) = 2$

1 $W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & \sin(2x) \\ \frac{1}{3}\sec(2x) & 2\cos(2x) \end{vmatrix} = -\frac{1}{3}\tan(2x)$

1 $W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} \cos(2x) & 0 \\ -2\sin(2x) & \frac{1}{3}\sec(2x) \end{vmatrix} = \frac{1}{3}.$

2 $u_1' = \frac{W_1}{W(y_1, y_2)} = \frac{-\frac{1}{3}\tan(2x)}{2} = -\frac{1}{6}\tan(2x)$; $u_2' = \frac{W_2}{W(y_1, y_2)} = \frac{\frac{1}{3}}{2} = \frac{1}{6}.$

$u_1 = -\frac{1}{6}\int \tan(2x)\,dx = -\frac{1}{6}\int \frac{\sin(2x)}{\cos(2x)}\,dx$ $\quad v = \cos(2x)$, $dv = -2\sin(2x)\,dx$

2 $= \frac{1}{12}\int \frac{dv}{v} = \frac{1}{12}\ln|v| = \frac{1}{12}\ln|\cos(2x)|$

$u_2 = \frac{x}{6}.$

1 $y_p = u_1y_1 + u_2y_2 = \frac{1}{12}\ln|\cos(2x)|\cos(2x) + \frac{x}{6}\sin(2x)$

$y(x) = y_h + y_p = c_1\cos(2x) + c_2\sin(2x) + \frac{1}{12}\ln|\cos(2x)|\cos(2x) + \frac{x}{6}\sin(2x)$