MTH 204 Name Key.

Quiz 4 Sec B.

6 Oct 2006

Follow the directions carefully. Please write neatly in pencil and show all your work. If you get stuck, feel free to ask me for help.

LEAD - Thursdays 5-7PM
CSF G5D.

Use variation of parameters to find a particular solution y_p to the DE

$2y'' - 12y' + 20y = 3e^{3x}\csc(x)$

$2y'' - 12y' + 20y = 0$ Assume $y(x) = e^{rx}$

$\Rightarrow e^{rx}(2r^2 - 12r + 20) = 0$

$\Rightarrow r = \frac{12 \pm \sqrt{144 - 4(2)(20)}}{2(2)} = \frac{12 \pm \sqrt{-16}}{4} = 3 \pm i$

$\begin{cases} y_1(x) = e^{3x}\cos(x) \\ y_2(x) = e^{3x}\sin(x) \end{cases}$

Put DE in standard form: $y'' - 6y' + 10y = \frac{3}{2}e^{3x}\csc(x)$

Assume $y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$

$$W(y_1, y_2) = \begin{vmatrix} e^{3x}\cos(x) & e^{3x}\sin(x) \\ 3e^{3x}\cos(x) - e^{3x}\sin(x) & 3e^{3x}\sin(x) + e^{3x}\cos(x) \end{vmatrix}$$

$$= e^{3x}\cos(x)[3e^{3x}\sin(x) + e^{3x}\cos(x)] - e^{3x}[3e^{3x}\cos(x) - e^{3x}\sin(x)]\sin(x)$$

$$= e^{6x}$$

$$u_1'(x) = \frac{W_1}{W(y_1, y_2)} = \frac{\begin{vmatrix} 0 & e^{3x}\sin(x) \\ \frac{3}{2}e^{3x}\csc(x) & - \end{vmatrix}}{e^{6x}} = \frac{-\frac{3}{2}e^{6x}}{e^{6x}} = -\frac{3}{2}$$

$$u_2'(x) = \frac{W_2}{W(y_1, y_2)} = \frac{\begin{vmatrix} e^{3x}\cos(x) & 0 \\ - & \frac{3}{2}e^{3x}\csc(x) \end{vmatrix}}{e^{6x}} = \frac{\frac{3}{2}e^{6x}\cot(x)}{e^{6x}} = \frac{3}{2}\cot(x)$$

$\Rightarrow u_1(x) = -\frac{3}{2}x$ $\quad u_2(x) = \frac{3}{2}\int\cot(x)\,dx = \frac{3}{2}\ln|\sin(x)|$

Then $y_p = u_1y_1 + u_2y_2$

$$= -\frac{3}{2}xe^{3x}\cos(x) + \frac{3}{2}\ln|\sin(x)|e^{3x}\sin(x)$$