MTH 204
Quiz 4
24 Feb 2006

Name: Key
Section: D/E

Follow the directions carefully. Remember to show all your work in order to get full credit. Please write neatly in pencil. If you have any trouble, feel free to ask me for help.

LEAD Session - Wednesdays
6:00 - 7:30 PM.
CSF G5D.

1. Find the smallest differential operator that annihilates the following functions.

$a(x) = 7 + xe^{2x}$ $\quad r = 0, 2, 2$

$r(r-2)^2 \Rightarrow D(D-2)^2$

$b(x) = \cos(2x) + 10e^{-x}\sin(3x)$ $\quad r = \pm 2i, -1 \pm 3i$

$(r^2+4)((r+1)^2+9) \Rightarrow (D^2+4)((D+1)^2+9)$

$c(x) = 6x^2 + 5x^3 + x\sin(3x)$ $\quad r = 0, 0, 0, 0, \pm 3i, \pm 3i$

$r^4(r^2+9)^2 \Rightarrow D^4(D^2+9)^2$

$d(x) = 5e^{3x} + 2e^{x} + 12$ $\quad r = 3, 1, 0$

$r(r-1)(r-3) \Rightarrow D(D-1)(D-3)$

$e(x) = 2e^{-7x} - \cos(5x)$ $\quad r = -7, \pm 5i$

$(r+7)(r^2+25) \Rightarrow (D+7)(D^2+25)$

<u>Hint</u>: What are the three types of annihilators we looked at?

$r = 0$ mult n $\Rightarrow D^n$

$r = \alpha$ mult n $\Rightarrow (D-\alpha)^n$

$r = \alpha \pm i\beta$ mult n $\Rightarrow [(D-\alpha)^2 + \beta^2]^n$

2. Solve the differential equation $y''+y'+y=3xe^x+10\cos(2x)$

Solve $y''+y'+y=0$ Assume $y(x)=e^{rx}$

$\Rightarrow e^{rx}(r^2+r+1)=0$

$$r^2+r+1=0$$

$$r=\frac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}=\frac{-1\pm\sqrt{-3}}{2}=\underbrace{-\frac{1}{2}}_{\alpha}\pm\underbrace{\frac{\sqrt{3}}{2}}_{\beta}i$$

$$\left.\begin{aligned} y_1(x)&=e^{-\frac{1}{2}x}\cos\left(\tfrac{\sqrt{3}}{2}x\right)\\ y_2(x)&=e^{-\frac{1}{2}x}\sin\left(\tfrac{\sqrt{3}}{2}x\right)\end{aligned}\right\} \quad y_h(x)=e^{-\frac{1}{2}x}\left[c_1\cos\left(\tfrac{\sqrt{3}}{2}x\right)+c_2\sin\left(\tfrac{\sqrt{3}}{2}x\right)\right]$$

$g(x)=3xe^x+10\cos(2x)$ $\quad r=1,1,\pm 2i$

$$(r-1)^2(r^2+4) \Rightarrow (D-1)^2(D^2+4)$$

$(D^2+D+1)y=3xe^x+10\cos(2x)$

$\Rightarrow (D-1)^2(D^2+4)(D^2+D+1)y=(D-1)^2(D^2+4)[3xe^x+10\cos(2x)]=0$

Assume $y(x)=e^{rx}$

$\Rightarrow (r-1)^2(r^2+4)(r^2+r+1)=0$

$$r=-\tfrac{1}{2}\pm\tfrac{\sqrt{3}}{2}i,\ \pm 2i,\ 1,\ 1.$$

$$y(x)=\underbrace{e^{-\frac{1}{2}x}\left[c_1\cos\left(\tfrac{\sqrt{3}}{2}x\right)+c_2\sin\left(\tfrac{\sqrt{3}}{2}x\right)\right]}_{y_h}+\underbrace{c_3\cos(2x)+c_4\sin(2x)+c_5e^x+c_6xe^x}_{y_p}$$

$y_p(x)=A\cos(2x)+B\sin(2x)+Ce^x+Exe^x$

$y_p'(x)=-2A\sin(2x)+2B\cos(2x)+Ce^x+Ee^x+Exe^x$

$y_p''(x)=-4A\cos(2x)-4B\sin(2x)+Ce^x+2Ee^x+Exe^x$

$$y_p'' + y_p' + y_p = (-4A+2B+A)\cos(2x)$$
$$+(-4B-2A+B)\sin(2x)$$
$$+(E+E+E)xe^x$$
$$+(C+2E+E+C+C)e^x$$
$$= 3xe^x + 10\cos(2x)$$

$\Rightarrow$ $-3A+2B=10$

$-2A-3B=0 \qquad \Rightarrow A = -\frac{3}{2}B$

$3E=3 \qquad \Rightarrow E=1$

$3C+3C=0 \qquad C=-1$

$\Rightarrow -3\left(-\frac{3}{2}B\right)+2B = \frac{13}{2}B = 10 \qquad \Rightarrow B = \frac{20}{13}$

$A = \frac{-30}{13}$

$$y_p(x) = \frac{-30}{13}\cos(2x) + \frac{20}{13}\sin(2x) - e^x + xe^x$$

So $y(x) = y_h + y_p$

$$= e^{-1/2x}\left[c_1\cos\left(\frac{\sqrt{3}x}{2}\right) + c_2\sin\left(\frac{\sqrt{3}x}{2}\right)\right] - \frac{30}{13}\cos(2x) + \frac{20}{13}\sin(2x) - e^x + xe^x$$