

MTH 204
Quiz 4
23 Feb 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.

Write neatly in pencil and show all your work
(you will only get credit for what you put on paper).

You may use your homework solutions.

Please do not share calculators during the quiz.

If you get stuck, feel free to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Find the general solution to the differential equation $2y'' + 6y' + 4y = \frac{4}{1+e^x}$.

Put DE in standard form: $y'' + 3y' + 2y = \frac{2}{1+e^x}$

Solve $y'' + 3y' + 2y = 0$. Assume $y(x) = e^{rx}$

$$\Rightarrow e^{rx}(r^2 + 3r + 2) = 0$$

$$(r+1)(r+2) = 0$$

$$\Rightarrow r = -1, -2$$

$$\Rightarrow y_h = c_1 e^{-x} + c_2 e^{-2x}$$

Assume $y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$

$$W(y_1, y_2) = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$$

$$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{2}{1+e^x} & -2e^{-2x} \end{vmatrix} = \frac{-2e^{-2x}}{1+e^x}$$

$$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{2}{1+e^x} \end{vmatrix} = \frac{2e^{-x}}{1+e^x}$$

$$u_1' = \frac{W_1}{W(y_1, y_2)} = \frac{\frac{-2e^{-2x}}{1+e^x}}{-e^{-3x}} = \frac{2e^x}{1+e^x}$$

$$u_2' = \frac{W_2}{W(y_1, y_2)} = \frac{\frac{2e^{-x}}{1+e^x}}{-e^{-3x}} = \frac{-2e^{2x}}{1+e^x} \leftarrow \text{Improper fraction}$$

$$u_1 = 2 \int \frac{e^x}{1+e^x} dx = 2 \ln|1+e^x|$$

$v = 1+e^x$
 $dv = e^x dx$

$$e^x + 1 \sqrt{\frac{-2e^x}{-2e^{2x} - 2e^x}} = \frac{-2e^x}{2e^x}$$

$$u_2 = 2 \int \left(-e^x + \frac{e^x}{1+e^x} \right) dx = -2e^x + 2 \ln|1+e^x|$$

So $y_p = u_1 y_1 + u_2 y_2$

$$= 2 \ln|1+e^x| e^{-x} + (-2e^x + 2 \ln|1+e^x|) e^{-2x}$$

$$= 2 \ln|1+e^x| e^{-x} + 2 \ln|1+e^x| e^{-2x} - 2e^{-x}$$

$$= (2e^{-x} + 2e^{-2x}) \ln|1+e^x|$$

$-2e^{-x}$
absorbed into y_1

Then $y(x) = y_h + y_p$

$$= c_1 e^{-x} + c_2 e^{-2x} + (2e^{-x} + 2e^{-2x}) \ln|1+e^x|$$