

MTH 204

Quiz 5

13 Oct 2006

Name: Key

Sec B

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
in order to get full credit. If
you get stuck, feel free to
ask me for help.

LEAD: Thursdays, 5-7 PM
CSF GSD.

Test 3: 27 Oct.

1. Solve the DE $3x^2y'' + 6xy' + y = 0$

Assume $y(x) = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$\Rightarrow 3x^2[m(m-1)x^{m-2}] + 6x[mx^{m-1}] + x^m = 0$$

$$x^m[3m^2 - 3m + 6m + 1] = 0$$

$$\Rightarrow 3m^2 + 3m + 1 = 0$$

$$m = \frac{-3 \pm \sqrt{9 - 4(1)(3)}}{2(3)} = \frac{-3 \pm \sqrt{-3}}{6} = -\frac{1}{2} \pm \frac{\sqrt{3}}{6}i$$

$$\text{So } y(x) = x^{-1/2} \left[c_1 \cos\left(\frac{\sqrt{3}}{6} \ln x\right) + c_2 \sin\left(\frac{\sqrt{3}}{6} \ln x\right) \right]$$

2. Solve the IVP $x^2y'' - 3xy' + 4y = 0$ $\begin{cases} y(1) = 2 \\ y'(1) = 1 \end{cases}$

Assume $y = x^m$

$$\Rightarrow x^2[m(m-1)x^{m-2}] - 3x[mx^{m-1}] + 4x^m = 0$$

$$\Rightarrow x^m[m^2 - m - 3m + 4] = 0$$

$$(m^2 - 4m + 4) = (m-2)^2 = 0 \Rightarrow m_1 = m_2 = 2$$

$$y(x) = c_1x^2 + c_2x^2 \ln x$$

$$y(1) = c_1(1) + c_2(0) = 2 \Rightarrow c_1 = 2$$

$$\Rightarrow y(x) = 2x^2 + c_2x^2 \ln x$$

$$y'(x) = 4x + 2c_2x \ln x + c_2x$$

$$y'(1) = 4(1) + 0 + c_2(1) = 1 \Rightarrow c_2 = -3$$

$$\text{So } y(x) = 2x^2 - 3x^2 \ln x$$

Bonus (2 points): Should the school change its name? If so, to what?