MTH 204

Quiz 5

3 March 2006

Name: Key

Section: D\E

Follow the directions carefully. Please write neatly in pencil. You must show all your work in order to receive full credit. If you have trouble, feel free to ask me for help.

LEAD Session - Wednesdays
6:00-7:30
CSF G5D.

Test 3: 24 March
4.5-

1. Consider the DE $y''-2y'+5y=e^x\sin(x)$

a. Classify the DE.

2nd order, linear, nonhomogeneous, constant coefficients

b. What method(s) do you have to solve the DE?

MUC ✓ or VOP

c. Solve the DE.

$y''-2y'+5y=0$ Assume $y(x)=e^{rx}$

$\Rightarrow r^2-2r+5=0$

$$r=\frac{-(-2)\pm\sqrt{4-5(4)}}{2(1)}=\frac{2\pm 4i}{2}=\underset{\alpha}{1}\pm \underset{\beta}{2}i$$

$\Rightarrow y_h=e^x[c_1\cos(2x)+c_2\sin(2x)]$.

$g(x)=e^x\sin(x)$ $\quad r=1\pm i \Rightarrow (r-1)^2+1=0$

$\Rightarrow \text{Ann}: [(D-1)^2+1]$

$(D^2-2D+5)y=e^x\sin(x)$

$\Rightarrow [(D-1)^2+1](D^2-2D+5)y=[(D-1)^2+1](e^x\sin(x))=0$ Assume $y(x)=e^{rx}$

$\Rightarrow [(r-1)^2+1](r^2-2r+5)=0$

$r=1\pm 2i,\ 1\pm i$

$$\Rightarrow y(t)=\underbrace{e^x[c_1\cos(2x)+c_2\sin(2x)]}_{y_h}+\underbrace{e^x[c_3\cos(x)+c_4\sin(x)]}_{y_p}$$

$y_p=e^x[A\cos(x)+B\sin(x)]$

$y_p'=e^x[A\cos(x)+B\sin(x)]+e^x[-A\sin(x)+B\cos(x)]$

$y_p''=e^x[A\cos(x)+B\sin(x)]+2e^x[-A\sin(x)+B\cos(x)]+e^x[-A\cos(x)-B\sin(x)]$

$=-2Ae^x\sin(x)+2Be^x\cos(x)$

$$y_p'' - 2y_p' + 5y_p = (2B - 2A - 2B + 5A)e^x\cos(x) + (-2A - 2B - 2(-A) + 5B)e^x\sin(x)$$
$$= 3Ae^x\cos(x) + 3Be^x\sin(x)$$
$$= e^x\sin(x)$$

$\Rightarrow A = 0$

$B = 1/3$

$$y_p = \frac{1}{3}e^x\sin(x)$$

Then $y = y_h + y_p$

$$= e^x[c_1\cos(2x) + c_2\sin(2x)] + \frac{1}{3}e^x\sin(x)$$

2. Consider the DE $4y''+36y=\csc(3x)$

a. Classify the DE.

2nd order, linear, constant coefficients, nonhomogeneous

b. What method(s) do you have to solve the DE?

VOP

c. Solve the DE. Sec 4-6 Example 2.

$y''+9y=\frac{1}{4}\csc(3x)$

Solve $y''+9y=0$ Assume $y(x)=e^{rx}$

$\Rightarrow r^2+9=0$

$r=\pm 3i$

$\Rightarrow y_1=\cos(3x)$

$y_2=\sin(3x)$

$$W(y_1,y_2)=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} = 3.$$

Assume $y_p=u_1y_1+u_2y_2$

$$\text{Then } W_1=\begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & \sin(3x) \\ \frac{1}{4}\csc(3x) & 3\cos(3x) \end{vmatrix} = -\frac{1}{4}$$

$$W_2=\begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} \cos(3x) & 0 \\ -3\sin(3x) & \frac{1}{4}\csc(3x) \end{vmatrix} = \frac{\cos(3x)}{4\sin(3x)}$$

$$u_1'=\frac{W_1}{W(y_1,y_2)}=\frac{-1/4}{3}=\frac{-1}{12} \Rightarrow u_1=-\int\frac{1}{12}dx=\frac{-x}{12}$$

$$u_2'=\frac{W_2}{W(y_1,y_2)}=\frac{\cos(3x)}{12\sin(3x)} \Rightarrow u_2=\frac{1}{12}\int\frac{\cos(3x)dx}{\sin(3x)} \qquad v=\sin(3x), \quad dv=3\cos(3x)dx$$

$$=\frac{1}{36}\ln|\sin(3x)|$$

$$y_p=\frac{-1}{12}x\cos(3x)+\frac{1}{36}\sin(3x)\ln|\sin(3x)|$$

$$\Rightarrow y=c_1\cos(3x)+c_2\sin(3x)-\frac{1}{12}x\cos(3x)+\frac{1}{36}\sin(3x)\ln|\sin(3x)|$$