Read the directions carefully.
Write <u>neatly</u> in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.
Please do not share calculators during the quiz.
If you get stuck, feel free to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
       CSF G5D

Suppose a 160 lb person stretches a bungee cord on the New River Gorge Bridge 200 ft. Then they are displaced an additional 100 ft before the cord is released. Assume there is no damping.

a. Set up and solve the initial value problem describing this motion. **Use** $g = 32 \text{ft} / s^2$ as the acceleration for gravity.

$$m = \frac{W}{g} = \frac{160}{32} = 5$$

$$\beta = 0$$

$$mg = Ks$$

$$160 = 200K$$

$$\Rightarrow K = \frac{4}{5}$$

$$\begin{cases} 5y'' + \frac{4}{5}y = 0 \\ y(0) = 100 \\ y'(0) = 0 \end{cases}$$

$$y(t) = e^{rt}$$

$$\Rightarrow e^{rt}\left(5r^2 + \frac{4}{5}\right) = 0$$

$$r^2 = -\frac{4}{25}$$

$$\Rightarrow r = \pm \frac{2}{5}i$$

$$\underbrace{\quad}_{\omega}$$

$$y(t) = c_1 \cos\left(\frac{2}{5}t\right) + c_2 \sin\left(\frac{2}{5}t\right)$$

$$y(0) = c_1(1) + c_2(0) = 100$$

$$\Rightarrow c_1 = 100$$

$$y(t) = 100\cos\left(\frac{2}{5}t\right) + c_2 \sin\left(\frac{2}{5}t\right)$$

$$y'(t) = -40\sin\left(\frac{2}{5}t\right) + \frac{2}{5}c_2 \cos\left(\frac{2}{5}t\right)$$

$$y'(0) = -40(0) + \frac{2}{5}c_2(1) = 0$$

$$\Rightarrow c_2 = 0$$

$$\text{So } y(t) = 100\cos\left(\frac{2}{5}t\right)$$

b. Now assume that the object is acted on by an external force of $f(t) = 20\cos(\gamma t)$. For what value of γ does resonance occur?

$$\omega = \gamma = \sqrt{\frac{K}{m}} = \sqrt{\frac{4/5}{5}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$