MTH 204
Quiz 5
20 Feb 2009

Name: Key

Section: C or F (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Consider the DE $y''-y'-2y=5e^{2x}+4\cos(x)$.

a. Classify the DE

2nd order, linear, CC, NH

b. What method(s) can we use to solve this DE?

{ MUC ✓
{ VOP

c. Solve the DE.

1. Solve $y''-y'-2y=0$, assume $y(x)=e^{rx}$

$\Rightarrow e^{rx}(r^2-r-2)=0 \Rightarrow (r-2)(r+1)=0$

$\Rightarrow r=2,-1$

$\Rightarrow y_h(x)=c_1e^{2x}+c_2e^{-x}$

2. Find roots of $g(x)$

Roots: $r=2, 0\pm i$

3. Find a characteristic equation for $g(x)$

$(r-2)(r^2+1)=0$

4. Find the smallest Ann of $g(x)$

$(D-2)(D^2+1)$

5. Apply Ann to both sides

$(D-2)(D^2+1)(D^2-D-2)y=(D-2)(D^2+1)g(x)=0$

5th order, linear, CC, Hom

6. Assume $y(x)=e^{rx}$

$\Rightarrow e^{rx}(r-2)(r^2+1)(r+1)(r-2)=0$

$\Rightarrow r=2,-1,0\pm i,2$

$y(x)=\underbrace{c_1e^{2x}+c_2e^{-x}}_{y_h(x)}+\underbrace{c_3\cos(x)+c_4\sin(x)+c_5xe^{2x}}_{y_p(x)}$

7. Identify $y_p(x) = A\cos(x) + B\sin(x) + Cxe^{2x}$

$y_p'(x) = -A\sin(x) + B\cos(x) + Ce^{2x} + 2Cxe^{2x}$

$y_p''(x) = -A\cos(x) - B\sin(x) + 4Ce^{2x} + 4Cxe^{2x}$

8. MUC

$y_p'' - y_p' - 2y_p = (-A-B-2A)\cos(x) + (-B+A-2B)\sin(x)$
$+ (4C - C)e^{2x} + (4C - 2C - 2C)xe^{2x}$

$= (-3A-B)\cos(x) + (A-3B)\sin(x) + 3Ce^{2x}$

$= 5e^{2x} + 4\cos(x)$

e^{2x}: $3C = 5 \Rightarrow C = \frac{5}{3}$

$\cos(x)$: $-3A - B = 4 \Rightarrow A = 3B = -\frac{6}{5}$

$\sin(x)$: $3(A-3B) = 0$

$-10B = 4 \Rightarrow B = -\frac{2}{5}$

9. GS

$y(x) = y_h(x) + y_p(x)$

$= c_1 e^{2x} + c_2 e^{-x} - \frac{6}{5}\cos(x) - \frac{2}{5}\sin(x) + \frac{5}{3}xe^{2x}$

Bonus (2pts): Find the smallest annihilator to

$g(x) = 6x^2 + 2 + 7xe^{3x} + \cos(x) + 5e^{2x}\cos(x)$

Roots: $r = 0, 0, 0, 3, 3, 0 \pm i, 2 \pm i$

Char Eq: $r^3(r-3)^2(r^2+1)[(r-2)^2+1] = 0$

Ann: $D^3(D-3)^2(D^2+1)[(D-2)^2+1]$