MTH 204
Quiz 6
28 Oct 2005

Name: Key
Section: A/C

Follow the directions carefully. Show all neatly in pencil.

Ends of Beam	Boundary conditions at $x=a$	
Embedded	$y(a)=0$	$y'(a)=0$
Free	$y''(a)=0$	$y'''(a)=0$
Hinged	$y(a)=0$	$y''(a)=0$

Test 3: 31 Oct, Monday.

LEAD Session - Mondays
6PM Rolla G4.

1. A beam of length L is embedded at its left end and free at its right end. Suppose the beam has a constant load w_0 that is uniformly distributed along its length ie $w(x) = w_0$, $0 < x < L$. Find the deflection $y(x)$ of the beam that satisfies the equation $EI\frac{d^4y}{dx^4} = w_0$.

$$\frac{d^4y}{dx^4} = \frac{w_0}{EI} \quad \begin{cases} y(0)=0 & y''(L)=0 \\ y'(0)=0 & y'''(L)=0 \end{cases}$$

$\frac{d^4y}{dx^4} = 0$ Assume $y(x) = e^{rx}$

$y'(x) = re^{rx}$

$y''(x) = r^2e^{rx}$

$y'''(x) = r^3e^{rx}$

$y^{(4)}(x) = r^4 = 0$

$r^4 = 0$

$r = 0$ mult 4.

$y_h(x) = c_1 + c_2x + c_3x^2 + c_4x^3$

Find annihilator of $\frac{w_0}{EI}$ $r=0$

$D\left[\frac{w_0}{EI}\right] = 0$

$DD^4y = D\left[\frac{w_0}{EI}\right] = 0$

$D^5y = 0$ Assume $y(x) = e^{rx}$

$\Rightarrow r^5 = 0$

$r = 0$ mult 5

$$y(x) = \underbrace{c_1 + c_2x + c_3x^2 + c_4x^3}_{y_h} + \underbrace{c_5x^4}_{y_p}$$

$y_p(x) = Ax^4$

$y_p'(x) = 4Ax^3$

$y_p''(x) = 12Ax^2$

$y_p'''(x) = 24Ax$

$y_p^{(4)}(x) = 24A$

$y_p^{(4)}(x) = \frac{w_0}{EI}$

$24A = \frac{w_0}{EI}$

$A = \frac{w_0}{24EI}$

$y(x) = c_1 + c_2x + c_3x^2 + c_4x^3 + \frac{w_0}{24EI}x^4$

$y'(x) = c_2 + 2c_3x + 3c_4x^2 + \frac{2w_0}{6EI}x^3$

$y''(x) = 2c_3 + 6c_4x + \frac{w_0x^2}{2EI}$

$y'''(x) = 6c_4 + \frac{w_0x}{EI}$

$y(0) = c_1 + 0 + 0 + 0 + 0 = 0$

$\Rightarrow c_1 = 0$

$y'(0) = c_2 + 0 + 0 + 0 = 0$

$\Rightarrow c_2 = 0$

$y''(L) = 2c_3 + 6c_4L + \frac{w_0L^2}{2EI} = 0$

$y'''(L) = 6c_4 + \frac{w_0L}{EI} = 0$

$\Rightarrow c_4 = -\frac{w_0L}{6EI}$

$$2c_3 + 6\left(\frac{-w_0 L}{6EI}\right)L + \frac{w_0 L^2}{2EI} = 0$$

$$2c_3 - \frac{w_0 L^2}{2EI} = 0$$

$$\Rightarrow c_3 = \frac{w_0 L^2}{4EI}$$

So $y(x) = \frac{w_0 L^2}{4EI} x^2 - \frac{w_0 L}{6EI} x^3 + \frac{w_0}{24EI} x^4$